An Examination of Cut-Off Rates for Capital Expenditure Analysis under Capital Rationing

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1. INTRODUCTION

The significance of capital expenditure analysis

(1) How much money will be needed for expenditures in the coming period? (2) How much money will be available? (3) How should the available money be doled out to candidate projects?[1]

In these words, Joel Dean has described capital budgeting, this term being defined as the planning of capital expenditures.[2] Capital expenditure analysis is one facet of capital budgeting. Question one refers to the demand for capital to finance capital investment proposals that have been submitted to management for its consideration. Question two refers to the various sources of funds which the firm has at its disposal to meet these investment proposals. It is the third question however, which is "the crux of the budgeting problem, the point where it becomes evident how much should be spent in total and where".[3] Once capital expenditure proposals are submitted for consideration, and the amount of resources available to finance them has been determined, management is still left with the task of answering this third question. To do this, a choice must be made between the projects competing for available funds.

Evaluations can be made of these proposals, and that project or group of projects which best fulfils the goals of management is selected. A method for evaluating proposals is to use a quantitative model. Benefits and costs applicable to the various alternatives are expressed in monetary terms, and these alternatives are then analysed to determine which one yields the greatest increase in corporate profits. This quantitative evaluation is called capital expenditure analysis.

1 Joel Dean, *Managerial Economics* (Englewood Cliffs, New Jersey: Prentice-Hall, 1951), p. 555.

2 A capital expenditure is defined as any outlay of funds which is expected to result in benefits accruing to the firm over a time period in excess of one year.

3 Dean, *Managerial Economics*, p. 555.

However, should this analysis reveal that a particular proposal produces the greatest increment in corporate profits, its acceptance does not automatically follow. Before making the final decision to accept a project, management must exercise judgment in balancing profitability against any qualitative factors inherent in the proposal. These could be so unfavourable to the firm that they outweigh the financial profitability of the project and lead to its rejection.

Nevertheless, the quantitative appraisal afforded by capital expenditure analysis is essential in project evaluation and selection for two reasons. Firstly, if a proposal is rejected for various qualitative reasons, some measure of the profit lost to the firm through this decision is provided. Secondly, because capital expenditure analysis requires quantification of the benefits and costs associated with a project, it forces management to consider various projects at some depth before a decision is made.

This second reason is extremely important, due to the nature of capital expenditures. By definition, a capital expenditure entails a current outlay in return for benefits to be derived in the future, but in waiting for these benefits to appear, the firm may have its capital tied up in a proposal for many years. Often capital may be recovered prematurely only at very high cost to the firm. The organization has lost some of its ability to adapt to future changes in business conditions, as a result of making a capital expenditure. In addition, capital expenditures are often subjected to a significant degree of risk, so the possibility always exists that anticipated benefits on which a decision to accept a project was made, may not materialize. Therefore, because of these characteristics of capital expenditure, it is most important that current decisions to invest in capital projects be made properly.

> With the growing complexity and highly specific character of modern production and marketing methods, investment decisions are becoming more irreversible. Thus investment decisions are increasingly determining the direction and pace of a company's future growth and limit the opportunities open to it in much the same way that the tracks determine the speed and directions open to a locomotive. This and the increasingly capital intensive nature of modern production methods necessitate careful consideration of the methods used for investment appraisal[4].

Since capital expenditure analysis forces a detailed consideration of many aspects of a proposal, it can greatly assist in making the correct investment decision.

Capital expenditure analysis and capital rationing

For any individual firm, a situation could arise in which it either cannot or does not wish to raise sufficient funds for all capital expenditure proposals deemed profitable. In this situation, the firm is under a condition of capital rationing. Various empirical studies have revealed that such a state of affairs could be quite prevalent in the business community. A survey in the United Kingdom in 1964[5] indicated that over 50% of the 300 companies questioned were subjected to capital rationing, while in the United States, 80% of the respondents to a more recent survey[6] of 163 companies were constrained by varying degrees of capital rationing.

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4 A[nthony] J. Merrett and Allen Sykes, *Capital Budgeting and Company Finance* (London: Longmans, Green and Co., 1966), p.viii.

5 Reported in G[erald] H. Lawson, "Criteria To Be Observed in Judging a Capital Project – II", *Accountants Journal* 56 (June 1964): 271-73.

6 See Alexander A. Robichek, Donald G. Ogilvie, and John D. C. Roach, "Capital Budgeting: A Pragmatic Approach", *Financial Executive* 37 (April 1969): 32.

Perhaps of more interest to the Australian situation however, is research performed by Meredith in 1964.[7] In a survey of 285 Australian public companies, Meredith discovered that 61.2% experienced some form of capital rationing at that time. This capital rationing was experienced only occasionally by 47.6% and consistently by 13.6% of the sample.[8] It was also found that rationing of funds was more prevalent in medium size companies than in those of larger or smaller size.[9] Meredith argued that this was caused by small companies having the smallest volume of capital expenditures, and the largest companies having greater borrowing power than smaller enterprises. As a general conclusion he stated:

> Capital rationing is an important factor in the capital budgeting process of Australian public companies. The exact significance of capital rationing varies from company to company, depending on company industry and on company size.[10]

If the empirical evidence is a true indication of environmental conditions, then one would expect a discussion of capital rationing and its effects on project selection to be found in the body of literature concerned with capital expenditure analysis. However, only minor attention seems to have been paid to this problem. The only justifiable reason for this is that capital rationing does not exist at all in practice.

> With some notable exceptions ... the literature also disregards the case of capital rationing, usually on the ground that rationing ought not to exist when firms behave rationally (in the narrow economic sense).[11]

However, the empirical research referred to above would suggest that firms *do* ration capital, so to rely on capital expenditure analysis techniques which assume that unlimited funds are available may cause incorrect selections of projects to be made. In view of the significance of capital expenditure analysis in project appraisal, such a situation could be very harmful to the firm.

Aim of the study

This study will examine the effects of capital rationing on one special facet of capital expenditure analysis—the cut-off rate. The significance of the cut-off rate arises because it can be used to determine whether or not a project is profitable. By comparing the profitability of all projects with this rate, management can determine when it is uneconomical to continue making capital investments. A decline in the profitability of projects below this cut-off rate is usually a signal to halt expenditures.

The cut-off rate plays an important role in the administrative control of capital project selection also. Lower levels of management can screen proposals using this rate, and those projects that are not acceptable quantitatively are not submitted to top management for final approval.

In periods of slack investment demand, the cut-off rate can be used to avoid making marginal investments of low profitability. Funds are preserved for use when investment demand increases. However, in a capital rationing situation this would not occur since by definition the firm has too many profitable projects for available funds.

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7 G[eoffrey] G. Meredith, *Capital Rationing and the Determination of the Firm's Performance Standards for Capital Investment Analysis*, University of Queensland Department of Accountancy Paper, Vol. I, No. 4 (1965), p.114.

8 *Ibid.*, p.100.

9 *Ibid.*, p.99.

10 *Ibid.*, pp.99-100.

11 H. Martin Weingartner, "Capital Budgeting of Interrelated Projects: Survey and Synthesis", *Management Science* 12 (March 1966): 485.

Although most of the discussion of cut-off rates under capital rationing will centre on theoretical issues, some consideration will also be given to practical applications of the ideal cut-off rates derived, because capital rationing is reported to be a practical issue affecting decision makers in the environment.

However, it should be apparent from the previous section that this entire exercise would be pointless if capital rationing did not exist in practice. Meredith's research, carried out seven years ago, might not be currently applicable. To discover whether capital rationing is still experienced in Australia, a stratified, random sample of 172 listed Australian public companies was surveyed by this writer, during September, October, and November 1969. The results of this survey are incorporated in the body of the study.

Scope of the study

It has been necessary for conciseness to restrict the scope of this study in the following ways:

1. Only business organizations are referred to, and no mention is made of capital expenditure analysis by individuals.

2. Capital expenditure analyses presented here show quantitatively the most profitable course open to the firm. It is then left to management to weigh this profitability against any qualitative factors associated with particular projects to decide whether they should be accepted.

3. Only capital expenditure analysis methods known as "discounted cash flow" techniques are discussed. Excluded are such non-discounting methods as payback and the accounting rate of return. Discounted cash flow techniques are now generally accepted as the best methods available, mainly because the time value of money is taken into account.

> The discounted cash-flow method for capital budgeting recognizes that the use of money has a cost (interest), just as the use of a building or an automobile may have a cost (rent). A dollar in the hand today is worth more than a dollar to be received (or spent) five years from today. For instance, in the interim a dollar can be invested in a savings institution; the dollar would grow markedly during a five-year span because of the interest it would earn. *Because the discounted cash-flow method explicitly and routinely weighs the time value of money, it is the best method to use for long-range decisions.*[12]

4. Only capital expenditure proposals subjected to negligible or equal degrees of risk will be considered.[13] In this way, no adjustments need be made to the analysis of projects to compensate for risk.

5. The only capital investment projects to be considered are all independent and divisible. The acceptance of an independent project does not affect other projects' profitabilities or chances of acceptance. Therefore, projects which are mutually exclusive are not considered in this study. Divisible investment projects are those requiring identical initial outlays. Investments which are of uneven size are not considered.[14]

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12 Charles T. Horngren, *Cost Accounting: A Managerial Emphasis*, 2nd ed. (Englewood Cliffs, New Jersey: Prentice-Hall, 1967), p.442.

13 For a survey of the literature on capital expenditure analysis under conditions of risk, as well as other aspects of capital expenditure analysis, see Myles M. Dryden, "Capital Budgeting: Treatment of Uncertainty and Investment Criteria", *Scottish Journal of Political Economy* 11 (November 1964): 235-59.

14 Readers interested in the treatment of interdependent, indivisible projects under capital rationing should see Weingartner, "Capital Budgeting of Interrelated Projects", and H. Peter Holzer and John D. Forsyth, "Mathematical Programming and the Rationing of Capital: Last in a Series", *Cost and Management* 42 (March 1968): 21-24.

Outline of the study

In section II, the nature of capital rationing and its prevalence in Australia are discussed. From the survey carried out by the writer, it is found that capital rationing is experienced by a majority of listed Australian public companies. Some comparisons are made between this finding and those of Meredith. Capital rationing may be imposed either by factors external to the firm or by a desire of management to limit funds to a predetermined amount. The latter is found to be the most prevalent form of capital rationing. As a result of the findings of this survey, some evaluation of cut-off rates under capital rationing seems warranted.

In section III an evaluation of cut-off rates under capital rationing is commenced. Discounted cash flow techniques under the assumption of unlimited funds are described, and the notion of the cut-off rate is elaborated upon. Under conditions of unlimited funds, this cut-off rate is usually assumed to be the cost of capital. The predetermined cut-off rate is introduced and its value when all projects are not known in advance is demonstrated. The "conventional analysis" of cut-off rates under capital rationing is described and its limitations pointed out. Attention is then focused on one of these limitations, the effect that the goals of corporate profit maximization or ordinary shareholders' wealth maximization have on cut-off rates. This is achieved by analysing two-period investments under capital rationing.

In section IV this analysis of two-period investments is used as a basis for examining cut-off rates with multi-period projects. These are investments with economic lives in excess of two periods. The complexities caused by the frequency of capital rationing when evaluating these projects are discussed. Multi-period investments are thought to be more prevalent in practice than two-period investments, and data from the survey of listed Australian public companies are presented to show that the cut-off rates derived in this section are not used in practice.

In section V, general conclusions to this study are offered.

II. THE NATURE OF CAPITAL RATIONING AND ITS PREVALENCE IN AUSTRALIA

Introduction

Before any investigation of cut-off rates under capital rationing can be attempted, it will be helpful to clarify just what is meant by this term "capital rationing" and the forms that it can take in the business environment. Knowledge of the incidence of capital rationing in Australian companies is also worthwhile, since there is little need for a description of detailed theoretical techniques for dealing with this situation if its current impact in practice is slight.

Therefore, this section serves a twofold purpose. It discusses the meaning of the term "capital rationing" and the forms it can take, and in addition presents some of the results of a survey carried out by the writer into the incidence of capital rationing in listed Australian public companies.

The meaning of the term "capital rationing"

A "capital rationing" situation was defined previously[15] as one in which a firm either cannot or does not wish to raise sufficient funds for all capital expenditure proposals deemed profitable. Whilst these words "capital rationing" intuitively imply some manner of constraint or limitation on available funds, the meaning attributed to them in disciplines other than Finance could cause confusion.

15 See above, p.78.

One of the earliest instances of the use of the expression "capital rationing" was that of an economist, Albert G. Hart, in 1937. He referred to the limited ability of the firm to raise funds externally.

> Even tho [*sic*] there is no uncertainty of the terms of financing available, there may still be need for liquidity if those terms involve *capital rationing*: if, that is, the firm's ability to get in outside capital is limited otherwise than by the interest it must pay.[16]

Hart recognized that it is the incidence of factors other than the borrowing costs of finance that cause a firm to suffer capital rationing. However, by stressing an inability to raise funds externally, implying that the firm *wants* to obtain more finance, he overlooked the possibility that the firm may limit the volume of funds available of its own accord.

Since the writings of Hart, capital rationing has been discussed mainly by economists interested in cost-benefit analysis in government enterprises. The capital limitation problem here is a very real one, especially in those areas concerned with resource development projects, where many competing proposals vie for appropriations of funds from the central treasury. Two forms of capital rationing have been stipulated—"maximum rationing" and "specific rationing". This classification probably stems from the fund appropriation system in government, whereby departments of government are assigned a specified amount of resources to utilize in a financial year.

> Turning now to the word "rationing," there are at least two different ideas here. The first we might call "specific rationing"; in this case the decision-making agent has a certain amount of current funds which he must dispose of through investment in projects yielding future returns. In the other case, which we will call "maximum rationing," the decision-maker can invest any amount up to, but not in excess of, the maximum amount indicated. This implies that some other use for the current funds is recognized as having value. For individuals, the alternative to investment of current funds is consumption; for firms, the alternative might be distribution of funds to stockholders; for departments of corporations or agencies of government, the alternative would be return of funds to the general treasury. The maximum rationing situation is obviously more interesting, being the more rational (or less irrational) of the two limitations considered.[17]

However, even though the term "capital rationing" is used in this sense by these writers, it is not the meaning which it takes in this study. The definition given in section I implies that proposals under consideration are analysed to determine their profitability using capital expenditure evaluation techniques, whether discounted or not, and then those proposals deemed profitable are ranked against available funds to see whether there are sufficient finances available to accept all of them. "Specific rationing" as used by Hirshleifer, De Haven and Milliman implies that *all* funds allocated to a firm for capital expenditures *must* be spent, regardless of whether the object of the expenditure is profitable or not. This does not qualify as capital rationing, except to the extent that the funds limit is insufficient to meet all profitable capital expenditures. "Maximum rationing" suggests that funds may be spent on capital proposals up to the point where further expenditure would be no longer profitable.

16 Albert G. Hart, "Anticipations, Business Planning, and the Cycle", *Quarterly Journal of Economics* 51 (February 1937): 290-91.

17 Jack Hirshleifer, James C. De Haven, and Jerome W. Milliman, *Water Supply: Economics, Technology, and Policy* (Chicago: University of Chicago Press, 1960), pp. 169-70.

Any excess funds are then returned to a central treasury to be reallocated to other government activities.[18] Unless the limit to available funds is exceeded by the resources necessary to meet all profitable capital expenditures, this "maximum rationing" is not a case of capital rationing as defined here.

Therefore, capital rationing occurs where there are insufficient funds for all capital expenditure proposals deemed profitable. This shortage of funds may arise for two separate reasons. In the first instance, it may be imposed on the firm by external factors. Even though the firm wants to raise more funds to engage in profitable proposals, these funds cannot be obtained. This is called "external capital rationing".[19] Secondly, rationing may be imposed by a desire of management to limit the amount of funds available for capital expenditures, even though sufficient funds are available internally or can be raised by borrowing. This situation is called "internal capital rationing".[20] It was with this interpretation of the term "capital rationing" that an approach was made to the problem of gaining empirical evidence to determine its incidence in Australian public companies.

The prevalence of capital rationing in listed Australian public companies

The survey

During September, October, and November 1969, a survey of listed Australian public companies was carried out by the writer. The purpose of this survey was fourfold, being designed to examine:

1. Whether the companies in the survey experienced capital rationing; and if so,
2. whether capital rationing was imposed by management or by external parties.
3. Whether these companies use discounted cash flow techniques to analyse capital expenditure proposals; and
4. how these companies modify their capital expenditure evaluation techniques to deal with capital rationing.

The population chosen was the 1,395 public companies listed on Australian stock exchanges at 30 June 1969, excluding any whose headquarters are overseas. This population was divided into the following 7 strata, each representing an industry group:

Group 1 included all finance, banking, investment, and insurance companies. This stratum was given the general title "Finance Group".

Group 2 included all companies involved in the conversion of raw materials into an "intermediate" product to be used later in the manufacture of a consumers' good. An example would be companies manufacturing pig-iron or plastics. The only exception to this was the inclusion of automobile and tractor manufacturers. This stratum was given the title "Heavy Industry Group".

Group 3 companies produced consumer's products, for example electrical appliances, clothing, or foodstuffs. This group was called the "Light Manufacturing Group".

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18 "Maximum rationing" is adopted in French nationalized industries. Where the budget of one industry could only be exhausted by accepting unprofitable projects, its budget is shrunk and the excess turned over to another industry which has a profitable use for the funds. See Thomas Marschak, "Capital Budgeting and Pricing in the French Nationalized Industries", *Journal of Business* 33 (April 1960): 134-35.

19 Harold Bierman, Jr., and Seymour Smidt, *The Capital Budgeting Decision*, 2nd ed. (New York: Macmillan Co., 1966), p. 182.

20 *Ibid.*, p. 187.

Group 4 included pastoral companies and companies involved in extraction activities, other than mineral mining and oil drilling. This group was called the "Primary Industries Group".

Group 5 companies sold a product or a service to the public. Retail stores and television stations are examples. The general title of this stratum was "Trade and Service Group".

Group 6 included mining companies and was called the "Mining Group".

Group 7 companies were those engaged in oil drilling, exploration or extraction, but not oil refining. This group was called the "Oil Group".

With the population split up into these divisions a stratified random sample of 172 companies was selected, representing 12.3 % of the population of 1,395 companies. A questionnaire and covering letter were mailed to each of these companies during October 1969, and the replies received totalled 88. (A copy of this questionnaire appears in appendix II.) Approximately one month later, a follow-up letter was mailed to those companies which had not replied. Of these, 40 responded to the follow-up giving the survey a total response of 128 or 78 %. Not all these returns could be used, however, because many questions were incomplete or ambiguously answered, and some companies refused to answer. The *usable* response rate was 59.9 % or 102 questionnaires. The following table shows how the total response was divided.

TABLE 1

Summary of replies to the survey of listed Australian public companies

	No.
Usable replies	102
Ambiguous replies	14
Incomplete or refused to answer	12
Total	128

The answers given by each of the companies responding to the follow-up appeal were carefully noted. It was reasoned that they were representative of all the companies not responding to the initial letter and questionnaire, so their results were weighted, in each stratum, to account for those companies which did not respond to the follow-up. That some adjustment for non-response should be made is recognized by writers in the areas of sampling theory and practice.

A survey population can be regarded as made up of two strata: respondents and non-respondents. Assuming that the sample was selected randomly in the first place, the former sub-population is adequately sampled. The problem is how to sample the latter, how to obtain *some* information about the people who do not complete questionnaires. There are several possibilities which can be applied singly or in combination:

1. Follow-up requests, enclosing a copy of the questionnaire and covering letter, can be sent to the non-respondents. This invariably produces further returns and these not only increase the numbers available for analysis but can be used to improve the overall results. Suppose that 40 per cent of the sample members return the questionnaires in the first instance, that follow-up letters are sent to the remaining 60 per cent and that a further 15 per cent (of the total) return

questionnaires as a result. It is then reasonable to regard these 15 per cent as more representative of the non-response population than of the 40 per cent initial-response population; in other words, the figures for the 15 per cent group should be given a weight of 60 per cent in arriving at the over-all results. This is preferable to either (a) treating the initial 40 per cent as representative of the whole population and not bothering with follow-up attempts at all, or (b) combining the 15 per cent with the 40 per cent without giving them any extra weight (which would be tantamount to assuming that the still remaining non-respondents—45 per cent—are no closer in characteristics to those who responded to the follow-up effort than to those who responded in the first place).[21]

Therefore, the results presented are given on the basis of a total response of 172 or 100%, even though the actual usable response rate was only 59.9%. It is thought that more reliable statistics may be derived by using this procedure than if the unweighted total of replies was analysed.

Some survey data

In the survey questionnaire, companies were asked whether they suffered either consistently or occasionally from capital rationing. The question took the following form:

2. In any one capital budget period has the supply of funds been sufficient to finance all the acceptable (profitable) proposals submitted for consideration?
   (1) *Always* sufficient funds available for proposals.
   (2) *Occasionally* capital funds *NOT* sufficient for proposals.
   (3) *Never* sufficient funds for all proposals submitted.[22]

There were 150 or 87.2% of the respondents who indicated that they experienced capital rationing either occasionally or consistently. The other 22 companies in the sample stated that they did not suffer from any shortage of funds.

From this result, the 95% confidence interval for the percentage of *all* listed Australian public companies that suffer from occasional or consistent capital rationing was computed to be 81.8% to 91.2%. This means one can be 95% sure that the true population percentage falls inside these limits.[23]

A similar survey, conducted by Meredith in 1964, revealed that 61.2% of Australian public companies experienced capital rationing, either consistently or occasionally.[24] The result obtained in the present survey seems to be significantly different from that obtained by Meredith.

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21 C. A. Moser, *Survey Methods in Social Investigation* (London: Heinemann Educational Books, 1958), p. 182. The method advocated here is not the only one available, but was considered to be the most practicable for this survey.

22 This question was similar to that used previously by Meredith, *Capital Rationing*, p. 87.

23 In any sample aimed at estimating some unknown population parameter, the statistic derived from the sample will probably differ from this true population parameter. To find limits within which the researcher can determine that this true parameter exists, with a known degree of error, a confidence interval should be derived from the sample.

For a mathematical discussion of confidence intervals, reference may be made to the following: Moser, *Survey Methods in Social Investigation*, pp. 68-69; H. D. Brunk, *An Introduction to Mathematical Statistics*, 2nd ed. (Waltham, Massachusetts: Blaisdell Publishing Co. of Ginn and Co., 1965), pp. 174-87; and Frederick E. Croxton and Dudley J. Cowden, *Applied General Statistics*, 2nd ed. (London: Sir Isaac Pitman & Sons, 1962), pp. 648-80, *passim*.

24 Meredith, *Capital Rationing*, p. 98.

A cautionary note

Before proceeding further, a cautionary note should be given. Many respondents to the survey indicated that they did not use discounted cash flow techniques for capital expenditure analysis, and few of those that did appeared to compute their firm's cost of capital. Instead, reliance was placed upon non-discounting methods such as payback and the accounting rate of return for capital expenditure evaluation. Therefore, the situation could have arisen in which respondents indicated shortages of funds due to the high cost of raising finance. As will be explained later, [25] the borrowing costs of finance are an element in the firm's cost of capital and this is used to determine project acceptability when discounted cash flow methods are used. In *these* circumstances, high costs of finance may render many projects unprofitable, but they do not give rise to a condition of capital rationing.

Nevertheless, this survey was designed to discover whether firms experienced capital rationing *as defined*. It is permissible for companies stating that they did suffer from capital rationing to use their own criteria for proposal profitability analysis, provided they were short of sufficient funds to fulfil all the proposals that *they* deemed profitable. However, it should be remembered that, although between 81.8% and 91.2% of listed companies experienced capital rationing, this confidence interval might have been set somewhat lower if all the firms in the sample had used discounted cash flow techniques.

Survey data analysed by industry group and by company size

The data obtained from answers to the question concerning the incidence of capital rationing were also analysed by industry group, and the results of this analysis appear in table 2 on page 87. It seems that capital rationing is most prevalent in the Heavy Industry Group (92%), Trade and Service Group (100%), and the Mining Group (93·5%), since the percentage of firms experiencing occasional or consistent rationing in these groups exceeds the sample average.[26]

The sample data were also analysed by company size, and to facilitate this the sample was split into 7 groups, each group including companies with the same book value of assets. The results of this analysis are presented in table 3. It should be noted first however, that the data given are based on *unweighted* sample figures so the total number of companies in the table is 102, the usable number of replies to the survey. Since the sample was stratified on an industry group basis, it is thought that weighting these data on a company size basis would be fallacious.

Bearing this in mind, most companies whose total assets range between 25 and 50 million dollars or fall beneath 1 million dollars seem to experience capital rationing to a greater extent than firms in the other company size groups. Nevertheless, capital rationing appears to affect companies of all other sizes fairly evenly, and at a high level. This would seem to contradict the findings of Meredith who concluded that companies of medium size had a higher incidence of capital rationing than large or small companies.[27]

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25 See below, p. 90.

26 No confidence limits were derived for the individual strata as was done for the whole sample, since the number of companies in each stratum is too small for these limits to be very meaningful.

27 Meredith, *Capital Rationing*, p. 99.

TABLE 2

Data showing the prevalence of capital rationing — analysed by industry group

Reply	Industry group													
	Finance		Heavy industry		Light manu-facturing		Primary producers		Trade and service		Mining		Oil		Total	
	No.	%	No.	%	No.	%	No.	%	No.	%	No.	%	No.	%	No.	%
No capital rationing	5	26	2	8	11	23	2	28	0	0	1	6.5	1	17	22	12.8
Occasional or consistent rationing	14	74	23	92	37	77	5	72	51	100	15	93.5	5	83	150	87.2
Total	19	100	25	100	48	100	7	100	51	100	16	100	6	100	172	100

TABLE 3

Data showing the prevalence of capital rationing — analysed by company size

Reply	Company size													
	Over $50 million		$25 million to $50 million		$10 million to $25 million		$5 million to $10 million		$2.5 million to $5 million		$1 million to $2.5 million		Under $1 million		Total	
	No.	%	No.	%	No.	%	No.	%	No.	%	No.	%	No.	%	No.	%
No capital rationing	2	14	0	0	2	20	2	13	3	15.5	5	21	1	6	14	19
Occasional or consistent rationing	6	86	9	100	8	80	13	87	16	84.5	19	79	16	94	88	81
Total	8	100	9	100	10	100	15	100	19	100	24	100	17	100	102	100

Why do the results given so far appear to differ so markedly from those given by Meredith? Perhaps the following discussion may serve as an explanation. Meredith's research was performed at a time when the economy was still recovering from the effects of the "credit squeeze" of 1961, and was unaffected by the mineral boom which has had an undoubted influence on many parts of the current business community. Firms are now engaged in expansionary capital expenditure activities to a greater extent than they were then. This can be seen by considering table 4 which shows total new capital expenditures by private businesses since 1961.

TABLE 4

Total new capital expenditures by private businesses

(Money amounts in millions of dollars)

Year	$
1961	220.0
1962	215.6
1963	245.0
1964	245.4
1965	308.8
1966	388.7
1967	367.8
1968	404.2
1969	449.0

Source: Commonwealth Bureau of Census and Statistics, *Seasonally Adjusted Indicators 1969* (Canberra: Government Printing Office, 1969), p. 73.

As will be explained later,[28] where there is an imperfect market for funds like the Australian capital market, firms can experience conditions of capital rationing. This is especially evident where many demands are being made on this market for funds. Since more capital expenditures are being undertaken now than previously, resulting in increasing fund requirements, it seems reasonable to assume that a higher incidence of capital rationing could prevail in our present environment than that found by Meredith.

Conclusions

The survey data have indicated that capital rationing is experienced by 87.2% of the companies sampled and the 95% confidence interval for the percentage of *all* listed Australian public companies is 81.8% to 91.2%. In the remainder of this section, the total incidence of capital rationing is analysed to determine the extent to which it is imposed externally or internally.

External capital rationing

It is thought that external capital rationing is caused by imperfections in the capital market. Where the market for funds is purely competitive, the firm would be just one of a vast number of small enterprises requiring capital resources. This market is so vast compared with the size of any firm that borrowings can be made in unlimited amounts. No risks are incurred by the lender in advancing funds to borrowers and the future is known with absolute certainty. In such a situation, the firm is able to borrow at a constant rate of interest since no single firm can influence this market rate. Any surplus funds held by the firm may be loaned at this same constant interest rate. The market rate of interest becomes the firm's cost of capital.

> So long as one of the main assumptions of most of capital theory is granted, namely, that capital markets are perfect and that the decision-maker may borrow or lend without constraint at the (given) market rate of interest, then the correct value for the cost-of-capital was [*sic*] simply the market rate of interest.[29]

28 See below, p. 90.

29 Dryden, "Capital Budgeting", p. 245.

This market rate of interest is the cost of capital regardless of whether the firm obtains its funds through issues of shares or debentures.

> Note that . . . the cost of capital is equal to the rate of interest on bonds, regardless of whether the funds are acquired through debt instruments or through new issues of common stock. Indeed, in a world of sure returns, the distinction between debt and equity funds reduces largely to one of terminology.[30]

Capital markets are seldom purely competitive, however. There always seem to be imperfections inherent in the structure of such markets which make the goal of pure competition an unattainable one. These imperfections can cause the borrowing rate of interest to fluctuate and differ among firms. It may be possible to influence the market so that funds are obtained on very favourable terms. Large firms may be able to approach the market with this result, but smaller enterprises may only be able to obtain funds at costs which increase with each additional increment of borrowings. Lower risks associated with the activities of big well-established firms in stable industries, as opposed to higher risks with smaller firms, account for much of these divergencies.

In addition, the presence of an imperfect capital market causes difficulties in estimating the costs of capital for an individual firm. Different forms of finance, such as debentures and short-term loans, may have different explicit costs, while there is a problem in determining the costs of ordinary shares and retained earnings which do not have explicit costs.[31] However, once these costs have been discovered, it is possible to combine them into an overall weighted average cost of capital for the firm. This can be based on an ideal capital structure, with the costs of each source of finance weighted to reflect the extent of its presence in this capital structure. A weighted average approach to cost of capital such as this, is preferable to treating the costs of individual sources of funds separately.

> If we tried to associate each source of funds with a particular investment, we would have the chaotic situation in which a machine might be purchased with a rate of return of 3 percent because it was "financed with debt" during the month, while a machine offering a 20 percent return might be rejected next month because it would have to be "financed by a new issue of common stock." Although the cost of capital may change over time, its level at any one moment should not be dependent upon the current block of new financing.
>
> But there is still another reason why we should view the cost of capital as a joint cost—a cost of a mixture of debt and equity. While a firm does not often float issues of debt and common stock in combination, each issue of debt is nonetheless dependent upon some equity base . . . Our capital is made available on a "package deal" basis, and it is the future cash payments that we must make on the entire mixture of capital sources that constitute our cost of capital.[32]

As more and more finance is obtained from the capital market, the average cost of capital eventually rises for many firms. A supply curve for capital can be drawn similar to the SS curve in figure 1, but the exact shape varies according to the size of the company and the exact imperfections inherent in the capital market.

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30 Franco Modigliani and Merton H. Miller, "The Cost of Capital, Corporation Finance and the Theory of Investment", *American Economic Review* 48 (June 1958): 262.

31 Actual measurement of the costs of these different sources of finance is beyond the scope of this study, but interested readers may refer to Robert W. Johnson, *Financial Management*, 3rd ed. (Boston: Allyn and Bacon, 1966), pp. 275-91, and James C. Van Horne, *Financial Management and Policy* (Englewood Cliffs, New Jersey: Prentice-Hall, 1968), pp. 109-139.

32 Johnson, *Financial Management*, pp. 285-86.

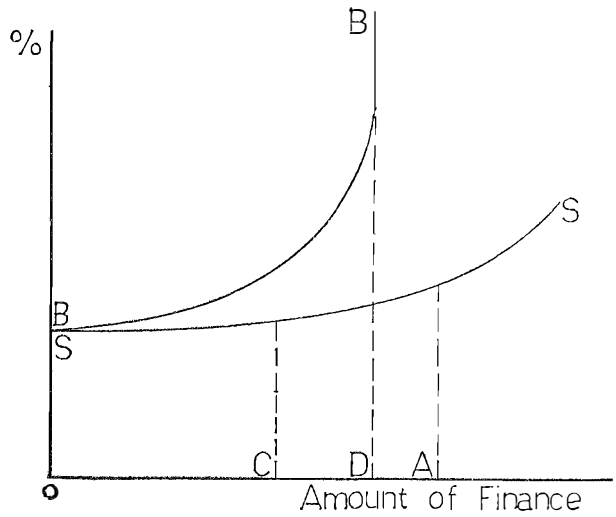

Fig. 1.—External capital rationing

Amounts of finance are shown on the x-axis and the average costs of this are shown on the y-axis. As will be seen later,[33] the cost of capital curve SS is used to determine the profitability of projects under consideration. However, it shall be assumed for the present that projects requiring OA funds have been deemed profitable. Obtaining OA funds necessitates borrowing from the capital market to add to those funds already possessed by the firm, which are shown on the diagram as OC. The volume of funds to be borrowed is therefore CA. However, beyond CD the firm is unable to obtain any funds at all and is facing an effective borrowing rate of interest which is infinitely high. This is shown by the borrowing curve for funds BB which becomes vertical at the limit of external borrowings. (The borrowing curve shows the costs of obtaining funds through issues of debt or equity.) The firm is then said to be under a condition of external capital rationing. Thus, although profitable projects would utilize funds up to OA, external capital rationing results in insufficient funds being obtained to engage in all of them.

The nature of external capital rationing having been discussed, consideration will now be given to its incidence in Australia, and to a discussion of some reasons underlying the market imperfections giving rise to this condition.

Incidence in Australia

External capital rationing seems to apply to some firms only, since many respondents to the survey indicated that they did not experience any difficulty in raising funds externally. The group surveyed were asked the following question:

7. Is capital rationing in your company
   (1) Caused by a desire of management to limit funds for capital expenditure proposals?
   (2) Caused by restrictions being placed on the Company by external bodies?
   (3) Caused by both these factors?[34]

33 See below, p. 103.

34 Further discussion of this question will be given in the section on internal capital rationing. See below, pp. 94-95.

There were 7 companies that checked part 2 of this question indicating that they experienced external capital rationing alone, while 61 respondents checked part 3, indicating that they suffered both internal and external rationing. Therefore, the total number of companies experiencing external capital rationing was 68, or 39.5% of the total sample.[35]

These results suggest that some inability to raise funds externally affects less than half of the companies sampled, and demonstrates that imperfections in the capital market only affect some firms. Others have no trouble obtaining external finance. These facts are brought to light more clearly when the data are analysed by industry group in table 5.

TABLE 5

The percentage of companies sampled suffering from external capital rationing

Industry Group	Percentage
Finance	47
Heavy Industry	40
Light Manufacturing	31
Primary Producers	14
Trade and Service	36
Mining	69
Oil	67

A higher than average incidence of external capital rationing appears to exist in the Mining and Oil Groups, while the other groups seem to experience this form of rationing to a lesser extent.

Some reasons for market imperfections and some empirical evidence

The higher than average incidence of external capital rationing in the Mining and Oil Groups can be explained by the unprecedented surge of activity in those areas over the last few years. The capital market is attracted to glamour capital issues in these areas, while companies with less exciting prospects are disregarded and their bids for new funds thwarted. This boom appears to have had little impact on the ability of firms in other areas to obtain funds, suggesting that this effervescence is being sustained only by the activation of investors' idle cash balances in saving banks, government bonds and the like. Resources do not appear to have been channelled away from the "industrial" section of the stock market to any extent.

On the other hand, a higher than average incidence of external capital rationing in the Mining and Oil Groups could reflect caution on the part of some lenders. Institutional lenders may refuse to advance loans to companies engaged in activities of doubtful profitability, such as mineral exploration, because the risk associated with the loan is very high. This is accentuated where the past performance of the borrower is poor.

> The poor profit record, however, may make it extremely costly if not totally impractical to raise further equity capital. Attempts to float a new equity issue at a time when the market is doubtful whether the firm can earn a worthwhile

35 The 95% confidence limits for the percentage of all listed Australian public companies suffering external capital rationing are 33.9% and 45.6%.

> return on the new equity may lead to a serious collapse in the price level of the firm's shares. At the same time, the poor record may also make it extremely difficult for the firm to obtain further debt capital.[36]

However, this would not be a case of capital rationing if the companies in question computed their costs of capital, since the risk inherent in a particular use of funds is a determining factor in this figure,[37] and the firm's cost of capital is used to determine project profitability. Nevertheless, for companies not computing their costs of capital, it is permissible to state that external capital rationing is suffered for this reason.

Even where the risk associated with a particular company's activities is not high, financial institutions may be unwilling to advance funds to a firm beyond a fixed limit, determined as a percentage of the book value of the company's assets.

> Nevertheless, the firm cannot go on borrowing indefinitely, because lenders always insist on a comfortable margin of safety and do not lend the full amount that the assets pledged as security are worth. Accordingly, the funds that the producer is able to borrow are limited to a certain proportion (or multiple) of his own funds invested in his enterprise.[38]

Inability to obtain funds may be caused by the size of the capital market relative to the size of companies which borrow from it. Some organizations needing large sums of capital to embark on major expansionary projects could discover that their requirements exceed the ability of the market to cater for them. Such a situation could arise in a developing nation where a large company has been formed to establish some basic industry, such as steel production. The capital requirements of this company could be so great that the market for funds is too small to handle it and the company faces a condition of external capital rationing. Again, even in a developed nation, "the flow of commercially viable projects resulting from technological progress may outweigh the ability of even the giant corporation to raise or generate finance".[39]

Frictions in the capital market are another form of market imperfection and may cause external capital rationing A firm which is seeking additional funds to finance profitable investment projects may discover that the market knows nothing about its capital needs, and strenuous efforts have to be made to find borrowers to advance funds. Alternatively, the firm and its fund requirements may be well known to borrowers, but there is a lag between the time the firm initially announces its intention to raise funds, and the time its requirements are fully satisfied. In both cases, a company may find itself temporarily in a situation of external capital rationing.

Other factors could cause the firm to suffer external capital rationing for considerable lengths of time. During periods of adverse economic conditions, general pessimism in the business community could result in an unwillingness to advance funds to borrowing companies, a situation lasting until the prospects for general prosperity brighten. Lenders who are usually willing to advance money during times of plenty are too cautious to risk their funds during recessions because they desire to keep a security buffer of liquid funds available to meet their own economic problems. Borrowers may also postpone their attempts to raise funds during these periods, and may be content to wait for a general improvement in the condition of the economy before going to the market again.

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36 A[nthony] J. Merrett and Allen Sykes, *The Finance and Analysis of Capital Projects* (London: Longmans, Green and Co., 1963), p. 140.

37 See Johnson, *Financial Management*, pp. 276-78, and Van Horne, *Financial Management and Policy*, pp. 136-39.

38 Tibor Scitovsky, *Welfare and Competition: The Economics of a Fully Employed Economy* (London: Unwin University Books, 1952), p. 200.

39 G[erald] H. Lawson and D. W. Windle, *Capital Budgeting and the Use of DCF Criteria in the Corporation Tax Regime* (Edinburgh and London: Oliver and Boyd, 1967), p. 56.

To see whether some of these market imperfections exist in Australia, companies in the survey were asked the following question:

8. If capital rationing is caused by external factors, are these
   (1) Difficulties in obtaining funds from
      (a) banks and short-term creditors
      (b) the issue of debentures
      (c) the issue of new shares.[40]

The results of data from this question are displayed in table 6.

TABLE 6

Some reasons for external capital rationing

Source of difficulty in obtaining funds	Number	Percentages	
		To total sample	To number of companies suffering external capital rationing
Banks and short-term creditors	36	20.9	52.9
The issue of debentures	2	1.2	2.9
The issue of shares	18	10.4	26.4

Notes: The 95% confidence limits for the population proportion suffering these difficulties in obtaining funds from these sources were:

(i) Banks and short-term creditors ..... 15.8% and 27.2%
(ii) The issue of debentures ..... 0.55% and 24.19%
(iii) The issue of shares ..... 6.9% and 15.5%

It can be observed that just over 50% of all companies suffering external capital rationing experience difficulties in obtaining bank loans and short-term credit. This could lend support to the earlier observation[41] that lenders advance funds only up to a certain proportion of the assets of the borrowing company. For instance, an application to a bank for an additional overdraft could be rejected because, in the opinion of the bank manager the borrower already had too much bank credit for the value of its assets.

Imperfections caused by restrictions on the firm

Another form of imperfection in the capital market is restrictions imposed on individual companies by external parties. Equity holders and debenture holders can place these restrictions on the firm, but they can also be imposed by government authority.

The provisions of existing debenture trust deeds may prohibit the firm from issuing further debt, and may strictly govern the terms and nature of short-term credit available to the firm. Restrictions imposed by existing equity holders may occur in

40 This is part 1 of question 8 on the survey questionnaire. The remainder of this question and the results from it will be given later in the section. See below, p. 94.

41 See above, p. 92.

the case where the company is a wholly-owned subsidiary of another. The parent may enforce strict guidelines as to the amount of capital available for proposals, or it may expropriate large portions of the subsidiary's annual profits as dividends, leaving it devoid of any funds. The parent company could also forbid the subsidiary's obtaining capital externally, by equity or debt issues. A similar situation could arise where the enterprise under consideration is a division of a larger firm, or where the enterprise is controlled by a strong group of individual shareholders.

Where the company is a government public enterprise, or is a member of a nationalized industry, it is subjected to restrictions by governmental authority. A public utility, for example, has funds available for capital expenditures voted to it in annual appropriations by parliament. Even though the leaders of such organizations can lobby strongly for more funds, determination of the amount finally granted to them each year is beyond their control. In the case where a public utility is allowed to raise funds directly from the capital market, its ability to do so would be closely governed.[42]

Company law forbids some enterprises from gaining access to the capital market and thus limits the funds available to those generated internally. For example, the proprietary company is forbidden by law to offer shares to the public or to issue debentures.[43] A condition of external capital rationing could affect one of these companies to such an extent that it is forced to convert itself to public company status to obtain external capital. Proprietary companies were not included in the population of companies sampled, so no data are available to support these claims. However, companies in the sample were asked whether they experienced restrictions imposed by some external parties. The specific question was:

8. If capital rationing is caused by external factors, are these

. . . . . .

(2) Restrictions imposed on the firm by Government authority?
(3) Restrictions imposed by existing debenture trust deeds?

There were 8 respondents who indicated that they had difficulty obtaining funds because of restrictions imposed by debenture trust deeds. This represents 4.7% of the total sample and 11.7% of those companies experiencing external capital rationing. Fourteen companies, or 8.1% of the total sample, and 20.6% of those respondents experiencing external rationing, indicated that they were inflicted with restrictions from government authority.

Conclusions

External capital rationing is caused by imperfections in the capital market, the result of several factors which have been mentioned here. From the survey, this form of capital rationing seems to be imposed on a minority of Australian public companies, although in the Mining and Oil Groups its incidence is higher than average. In most cases, external capital rationing is an occurrence for which the firm can do little else except wait until its requirements for funds are met.

Attention is now directed to the other form of funds restriction—internal capital rationing.

Internal capital rationing

In the case of internal capital rationing, there are profitable projects available and a ready supply of funds to engage in them, but management stipulates that only a

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42 The structure of government finances is explained in W. R[obert] C. Jay and R[ussell] L. Mathews, eds., *Government Accounting in Australia: A Book of Readings* (Melbourne: F. W. Cheshire 1968), *passim.*

43 *Companies Act, 1961* (Queensland) sec. 15, 1 (c).

certain amount will be available in any year for capital expenditures. Some writers had guessed intuitively that this form of capital rationing was the most prevalent,[44] and the results of the survey seem to bear out their observations. One of the questions asked was:

7. Is capital rationing in your company:
   (1) Caused by a desire of management to limit funds for capital expenditure proposals?
   (2) Caused by restrictions being placed on the company by external bodies?
   (3) Caused by both these factors?[45]

There were 83 companies which checked part 1 of this question, demonstrating that they experienced internal capital rationing alone, while 61 companies checked part 3 of the question indicating that they suffered from both internal and external capital rationing. Thus a total of 144 respondents suffered from internal capital rationing. This represented 83.7% of the total sample and 95.3% of those companies experiencing capital rationing in all forms.[46]

The information derived from the sample was also analysed by industry group, and the results appear in table 7.

TABLE 7

The incidence of internal capital rationing: analysed by industry group

Industry group	Percentage of the sample under internal capital rationing %	Percentage of companies under internal capital rationing to those under rationing in all forms %
Finance	42	61
Heavy Industry	88	96
Light Manufacturing	77	97
Primary Producers	71	100
Trade and Service	100	100
Mining	94	100
Oil	84	100

Internal capital rationing seems to be very prevalent in all industry groups other than finance.

Methods of imposing internal capital rationing and reasons for its existence

Capital rationing may be imposed internally in four different ways. Firstly, there could be a stipulation by management that the firm will not borrow funds from the capital market, but will be content to finance projects from retained earnings. Internal capital rationing arises in this case where total internally generated funds are insufficient to finance all projects deemed profitable.

44 See Meredith, *Capital Rationing*, p. 90, Merrett and Sykes, *The Finance and Analysis of Capital Projects*, p. 140, and Gerald H. Lawson, "Capital Investment Criteria in Business—II", *Accountant* 148 (April 1963): 491.

45 This question was used earlier in the discussion of external capital rationing. See above, p. 90.

46 The 95% confidence limits for the proportion of the population that experiences internal capital rationing are 77.7% and 88.5%.

> For many firms it is a source of pride that they never go to the market for financing a new opportunity, no matter how profitable it appears. They determine the amount of capital available for new schemes not on the basis of earning prospects, but on the basis of availability of cash from retained earnings above the balance needed to meet the ironclad standards for the current ratio.[47]

On the other hand, if use of external funds is permitted, the type of borrowings made and their rate of occurrence are strictly governed. For instance, some firms may refuse to issue long-term debt, and may allow borrowing from banks only at certain times during the year.

Secondly, capital rationing arises where there are plenty of internally generated funds available, but management sets an upper limit to their use in capital expenditures. Often this takes the form of a limit equal to annual depreciation charges plus a proportion of current earnings less dividends payable, and in some firms, the limit may not vary from year to year. In others, the exact amount set depends on the availability of projects and their profitability.

> There are no hard-and-fast rules to be found in practice regarding the selection of an over-all constraint. The net present values (or some similar measure such as time-adjusted rate of return) may strongly influence the over-all budget amount. For example, a flock of projects with huge net present values would probably result in a much higher over-all budget than would a group of projects that all slightly exceeded zero.[48]

Thirdly, management may ration capital by setting a cut-off rate for project acceptability higher than the firm's cost of capital, or other minimum rate of profitability. This results in a smaller number of projects being accepted. Although this procedure does not predetermine the amount to be spent on capital proposals in any year, its use in examining projects achieves the same result.

Fourthly, capital limitations may be imposed by management on certain areas of the firm, or for certain classes of project, while the other capital expenditure activities are left intact. Management of a divisionalized enterprise may state that expansionary projects in one segment, or for the entire firm, are to be limited to a certain sum of money, and replacement proposals for the entire firm or a segment are cut short after a set volume of funds has been expended.

While the writer's survey of Australian public companies did not ascertain whether internal capital rationing was executed by any of the first three methods, it did attempt to discover whether this fourth procedure was in evidence in the environment. Companies in the sample were asked:

9. Does your company limit funds available for capital expenditure proposals in specific areas and not others?
   1. Yes
   2. No
   3. Sometimes

There were 45 respondents who indicated "Yes" to this question and 41 checked "Sometimes", making a total of 86 companies or 50% of the sample[49] that limit funds available to specific areas of their organizations at some time.

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47 Joel Dean, *Capital Budgeting: Top-Management Policy on Plant, Equipment, and Product Development* (New York: Columbia University Press, 1951), pp. 53-54.

48 Horngren, *Cost Accounting*, p. 497. "Net present value" is one of several discounted cash flow techniques which will be discussed in section III. See below, p. 100.

49 The 95% confidence limits for the proportion of the population imposing this form of capital rationing are 43% and 57%.

The companies experiencing this class of capital rationing were asked the following question, designed to narrow down the form in which this rationing was imposed.

10. If the answer to question 9 was "Yes" or "Sometimes," are funds limited in the case of:
    (1) Expansionary proposals Company-wide (e.g. buying a new factory)?
    (2) Replacement proposals Company-wide?
    (3) Expansionary proposals in a segment of the Company?
    (4) Replacement proposals in a segment of the Company?

The data obtained from this question are analysed in the following table.

TABLE 8

Internal capital rationing imposed on certain classes of investment

	Company-wide		Restricted in one segment	
	Expansionary proposals	Replacement proposals	Expansionary proposals	Replacement proposals
No. of companies	41	16	44	13
% of companies to total sample	23.8%	9.3%	25.6%	7.5%
% of companies to those limiting funds in specific ways	47.6%	18.6%	51.2%	15.0%
% of companies to those suffering internal rationing	28.4%	11.1%	30.5%	9.0%

It seems that several companies limiting funds in specific areas or for specific tasks do so for expansionary proposals. However, very few firms limit funds for replacement projects. Restrictions of funds to segments of the firm seem to be slightly more prevalent than limitations imposed company-wide. However, this difference is probably not statistically significant. It might be concluded that many companies restricting funds in specific areas, or for specific kinds of investment, do so in a manner different from that presented here. The survey did not ascertain how these other restrictions are imposed.

Internal capital rationing may be imposed in one year only, and thereafter all restrictions on capital expenditures are lifted. Where the goal of the firm is to maximize profits, internal rationing should be adopted only as a matter of expediency so that essential current payments can be maintained, or because large volumes of funds will be needed for expenditures in future years, and plans might be nullified if too many resources are committed to current projects. The company may also be entering into a "period of impending change in management personnel, when the status quo is maintained",[50] and temporary constraints are placed on capital expenditures. To the extent that capital rationing is imposed for these reasons, its existence is justified.

50 Horngren, *Cost Accounting*, p. 498.

However, internal capital rationing can be imposed over several periods of time also, and in this form it must be condemned, where profit maximization is the goal of the firm.

> There is no conceptual justification for such a budget ceiling. All projects that enhance long-run profitability should be accepted. This is the only decision rule that makes economic sense. To the extent that capital rationing exists, it should be a "short-run phenomenon, limiting expenditures only to the current year or two."[51]

Internal capital rationing causes some profitable proposals to be foregone, because it results from choosing funds for capital projects before an analysis is undertaken of the acceptability of all projects. To do this is "to put the cart before the horse".[52] Projects should be subjected to capital expenditure analysis first, *then* the necessary funds should be determined, and the firm's prosperity will be enhanced by engaging in all acceptable proposals.

Internal capital rationing arises where the profit maximization goal has been relaxed within the firm. This relaxation results from the degree of competence and financial sophistication in management, and its aversion to all forms of risk.

Firstly, consider one of the methods for inflicting internal capital rationing on a firm; that of segregation from, or limited access to, the capital market. Need for this arises because in many cases management lacks the skill to undertake all projects deemed desirable simultaneously. Rather than obtain the necessary experienced personnel externally, management shelves a project.

> The need for capital rationing may also arise where there is no effective funds constraint, but where the rate of growth must be restrained because of managerial "indigestion," or shortages of skilled personnel or critical materials.[53]

Secondly, management may display financial sophistication in not wanting to obtain funds externally, through the medium of a new share issue, for fear of diluting the equity of the existing shareholders.

Thirdly, abstention from the capital market can be caused by a desire of management to avoid the perils that this can involve. These include the risk of liquidation because of unfavourable future events, and the burden thrown on the company by the presence of debt in the capital structure. Management fears not only for the safety of the firm, but also for its own security, since this is in jeopardy if liquidation is a possibility in the future.

> Debt financing puts an asymmetrical risk on management. The men who make the decisions rarely regard the profit propects as adequate to offset the threat to their personal security from general reorganization in bankruptcy. In many corporations, management's share in the profits of successful ventures (in the form of dividends on the stock they own) is an insignificant source of income compared with their salaries, which show admirable stability over the business cycle.[54]

Even where financial disaster is unlikely, borrowing may be kept to a minimum because management fears the restrictions lenders may place on the firm in return for their money. Management could desire to maintain a reserve of borrowing power for use when an unexpectedly profitable proposal is discovered, or to use in emergencies.

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51 *Ibid.*, p. 497, quoting H. Martin Weingartner, "The Excess Present Value Index—A Theoretical Basis and Critique", *Journal of Accounting Research* 1 (Autumn 1963): 214.

52 Lawson, "Capital Investment Criteria in Business—II", 492.

53 G. David Quirin, *The Capital Expenditure Decision* (Homewood, Illinois: Richard D. Irwin, 1967), p. 176. That this state of affairs is very real in some organizations is argued in F. K[enneth] Wright, "Project Evaluation and the Managerial Limit", *Journal of Business* 37 (April 1964): 179-85.

54 Dean, *Managerial Economics*, p. 581.

On the other hand, where the company is effectively controlled by management, a fear may exist that this grip on control could be shaken. "There is a limit, therefore, to the amount of stock [management] can sell in the market and the amount of funds [they] can raise by selling stock *without risking the loss of control*".[55]

Fourthly, management may be trying to avoid upsets to normal methods of business. It is content to earn a satisfactory profit "considering safety and control to be more important than additional profits".[56] In addition, utilization of all profitable opportunities might entail too much effort, and management would prefer an easy life, so it contents itself with a limited capital budget.

Summary

Internal capital rationing is by far the most prevalent form of capital rationing among listed Australian public companies, being suffered by 83.7% of the total sample taken, and 95.3% of those companies experiencing capital rationing in all forms. It may be imposed either by restricting funds for projects to retained earnings, a proportion of retained earnings, by setting a high cut-off rate for project acceptability, or by restricting capital expenditures in certain areas of the firm.

Where the goal of the firm is to maximize profits, internal capital rationing is defensible if imposed as a matter of expediency in the short run. However, when this capital restriction is allowed to stifle acceptable projects over several years, the maximization goal is not achieved. In this situation, internal capital rationing should be discontinued.

Summary and conclusions

In this section it has been argued that a capital rationing situation is one in which the firm either cannot or does not wish to raise sufficient funds for all capital expenditure proposals deemed profitable. From a survey of listed Australian public companies carried out by the writer, it was found that capital rationing is experienced by a majority of these companies. The percentage of companies in the sample experiencing capital rationing was 87.2% and the 95% confidence limits for the percentage of *all* listed Australian public companies suffering from capital rationing were computed to be 81.8% and 91.2%.

Capital may be rationed in two ways. Firstly, the firm may experience external capital rationing where the funds available to the firm are limited by the capital market. This form of capital rationing affected 39.5% of the sample. Secondly, the firm may suffer from internal rationing where funds for capital expenditures are limited by management. This was found to be the most prevalent form of rationing, being experienced by 83.7% of the sample.

Therefore, the predominance of capital rationing among listed Australian public companies will be taken as established, and it seems than an examination of cut-off rates under these conditions is warranted. There is a need for a discussion of possible modifications to cut-off rates determined under the assumption of unlimited funds. This discussion is given in the remainder of this study.

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55 Scitovsky, *Welfare and Competition*, p. 197.

56 J. Fred Weston and Eugene F. Brigham, *Managerial Finance*, 2nd ed. (New York: Holt, Rinehart and Winston, 1966), p. 162.

III. THE CUT-OFF RATE UNDER CAPITAL RATIONING

Introduction

In the previous section, a conclusion was reached that an investigation of cut-off rates under capital rationing seems desirable. This discussion will be commenced by briefly explaining some discounted cash flow techniques under the assumption of unlimited funds, and the importance of the cut-off rate in using these techniques. Observing the nature of this cut-off rate provides a good introduction to the discussion of its behaviour when a capital restriction is imposed.

Discounted cash flow techniques

It was stated in section I[57] that discounted cash flow methods of capital expenditure analysis are superior to any others, because they take into account the time value of money. Simply stated, the time value of money means that a dollar today is worth more than a dollar to be received at some time in the future. This dollar in hand now can be invested to return more than a dollar in years to come. Therefore, in any investment decision, the timing of cash flows plays an important part in determining project acceptability.

Current and future dollars may be compared by reducing future dollars to present values.

> Expected future cash flows are discounted to present value by a discount rate; the present value of cash inflows is then compared with the present value of cash outflows. This procedure allows the firm to decide whether to accept or reject the investment proposals under consideration.[58]

However, there are several discounted cash flow criteria available which can be used to obtain these present values. The most important of these are three in number.

Net present value

With the net present value criterion, all cash benefits and outflows associated with a project are discounted to a common point in time (usually the current period), at a predetermined discount rate in each period. The assumption is usually made that this rate of discount is constant over the life of the project. Net present value is obtained by subtracting the present value of cash outflows from the present value of cash benefits to arrive at one figure representing absolute investment worth. If a project has a positive net present value and is accepted, an immediate increase in the future wealth of the firm results.

Mathematically, the net present value of an investment proposal is:

$$\text{Net present value} = \left[\frac{S_1}{1+i} + \frac{S_2}{(1+i)^2} + \cdots + \frac{S_n}{(1+i)^n}\right] - \left[C_0 + \frac{C_1}{1+i} + \cdots + \frac{C_n}{(1+i)^n}\right]$$

where C_0, C_1, ..., C_n are year-end cash outflows in periods 0, 1, 2, ..., n; S_1, S_2, ..., S_n are year-end cash benefits in periods 1, 2, ..., n.; and i is the appropriate constant discount rate.

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57 See above, p. 80.

58 Van Horne, *Financial Management and Policy*, p. 17.

Present value index

This criterion of investment worth is computed by dividing the present value of future *net* cash flows from a project by its initial outlay.[59] Mathematically

$$\text{Present value index} = \frac{\frac{S_1 - C_1}{1+i} + \frac{S_2 - C_2}{(1+i)^2} + \cdots + \frac{S_n - C_n}{(1+i)^n}}{C_o}$$

where $S_1, S_2, \ldots, S_n$; $C_o, C_1, \ldots, C_n$ and i are defined as before.

The present value index gives an absolute figure as a measure of investment worth, representing the present value of net cash flows per dollar of initial outlay.

The internal rate of return

This is the discount rate which equates the present value of cash outlays and the present value of cash benefits for an investment. In other words, it is the rate of discount that reduces the net present value of the proposal to zero. Mathematically, the internal rate of return is represented by the discount rate r in the following equation:

$$C_o + \frac{C_1}{1+r} + \cdots + \frac{C_n}{(1+r)^n} = \frac{S_1}{1+r} + \frac{S_2}{(1+r)^2} + \cdots + \frac{S_n}{(1+r)^n}$$

where $S_1, S_2, \ldots, S_n$, and $C_o, C_1, \ldots, C_n$ are defined as before.

The internal rate of return expresses a proposal's expected profitability as a single "rate of return per year" on the capital outstanding per period while it is still invested in the proposal.[60]

The cut-off rate

It is no use determining the net present value, present value index, or Internal rate of return for a proposal if the firm does not know how to use these criteria in ascertaining that proposal's acceptability. To decide whether the firm will undertake an investment or not, acceptance rules must be established with each of these three criteria. The appropriate acceptance rules have been summarized by Prest and Turvey:

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59 There appears to be some controversy about whether this is the correct definition of the present value index. Some writers define it as the present value of cash benefits divided by the present value of cash outflows. That is,

$$\text{Present value} = \frac{\frac{S_1}{1+i} + \frac{S_2}{(1+i)^2} + \cdots + \frac{S_n}{(1+i)^n}}{C_o + \frac{C_1}{1+i} + \cdots + \frac{C_n}{(1+i)^n}}$$

However, cash outlays on a project subsequent to its acceptance are likely to be financed by the cash inflows from the project itself. A firm under capital rationing is likely to be more interested in actual cash outflows from the firm in the form of initial outlays. Since the definition of present value index stated in the text places more emphasis on the initial outlay of a project than it does on the subsequent cash flows, it is preferred to the alternative definition. For a further discussion see Bernhard Schwab and Peter Lusztig, "A Comparative Analysis of the Net Present Value and the Benefit-Cost Ratio as Measures of the Economic Desirability of Investments", *Journal of Finance* 24 (June 1969): 507-516.

60 The internal rate of return may also be interpreted as a constant rate of return per year on capital *originally* invested in the proposal. This interpretation implicitly assumes that the cash benefits from the proposal can be reinvested in the firm at this same internal rate of return.

1. Select all projects where the present value of benefits exceeds the present value of costs [that is, where the net present value is positive];

2. Select all projects where the [present value index] exceeds unity;

. . . . . .

4. Select all proposals where the internal rate of return exceeds the chosen rate of discount.[61]

In each of these acceptance rules, reliance is placed on the "chosen rate of discount", whether in reducing the future cash flows to present values in computing the net present value and present value index, or in comparisons with the internal rate of return. This rate of discount is the cut-off rate. As stated in section I,[62] the cut-off rate is of great importance in capital expenditure analysis because it is a necessary factor in determining the point where management should cease spending funds on capital projects.

Where all competing investment proposals in a given year are divisible and known in advance, together with the costs of raising sufficient funds to finance them, obtaining the correct cut-off rate is a relatively simple matter. It is determined by the point at which the profitability of projects falls to the level of the cost of funds. This is illustrated in figure 2.

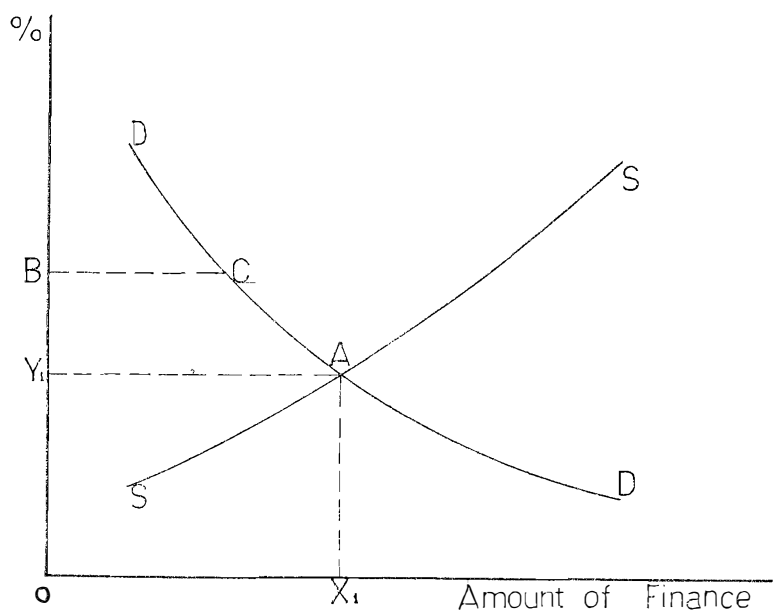

Fig. 2.—Cut-off rate under conditions of unlimited funds

The SS curve represents the cost of funds needed to finance proposals. This curve rises as more and more funds are raised, reflecting the fact that the firm is unable to

61 A. R. Prest and R[alph] Turvey, "Cost-Benefit Analysis: A Survey", *Economic Journal* 75 (December 1965): 703.

62 See above, p. 79.

obtain funds at a constant cost of capital. The cost of raising these funds is then compared with the profitabilities of competing proposals, the latter being shown in figure 2 by the DD curve. In order to equate the profitability of investments and the cost of funds for the investments, it is necessary to express profitability in terms of percentage rate of return. Therefore, the DD curve shows projects ranked in descending order, by their internal rates of return.

The intersection of the DD and SS curves at A determines the cut-off rate. At A, OX_1 funds would be invested in capital investments, and for each project the internal rate of return exceeds the cost of funds. To invest in projects beyond X_1 would contradict the profitability goal of the firm, because funds would then cost more than the return they provide. Therefore, OY_1 is the cut-off rate for a firm faced with these investment and financing opportunities.

By comparing OY_1 with the internal rate of return of a proposal it is immediately apparent whether this proposal is profitable. Project C is acceptable because its internal rate of return, OB, exceeds OY_1. However, is OY_1 the correct cut-off rate when using the net present value criterion? The answer has been provided in the following words:

> These two approaches [internal rate of return and net present value] give the same results for "accept or reject" decisions. This is so because the computed [internal rate of return] on a project will be higher than the cost of capital in all cases for which the present value of earnings discounted at the cost of capital is greater than the present value of outlays.[63]

Therefore OY_1 is the correct cut-off rate to use with net present value. Since the present value index uses the same discount rate as the net present value, OY_1 is also the correct cut-off rate for that criterion.

This universality of the cut-off rate OY_1 can be shown by considering figure 3. Part A of figure 3 shows the relationship between the discount rate displayed on the x-axis, and the net present value of a particular proposal drawn as the curve NPV. Part B of the figure shows the relationship between the discount rate drawn on the x-axis, and the present value index of this same proposal displayed as the curve PVI.

At the point R in part A, the net present value of the proposal is zero and OR is the proposal's internal rate of return, since this is the point at which the present value of its benefits equals the present value of its costs, and their algebraic sum is zero.[64] Similarly, at point R′ in part B, the present value of future net cash flows equals the initial outlay on the project and the present value index is unity. R′ is the internal rate of return also.

For discount rates less than R in part A, the net present value exceeds zero signifying project acceptance. Similarly, in part B, for discount rates less than R′, the present value index exceeds unity, also signifying project acceptance. In both cases the discount rate is less than the internal rate of return, and this too indicates acceptability of the proposal.

If the cut-off rate determined in figure 2[65] is shown as Y_1 and Y_1' in figure 3, the particular proposal shown in figure 3 will be accepted if either of the three investment criteria are used. Therefore, this cut-off rate is valid for use with all three of these criteria.

63 Ezra Solomon, "The Arithmetic of Capital-Budgeting Decisions", *Journal of Business* 29 (April 1956): 124-25.

64 This was stated earlier. See above, p. 101.

65 See above, p. 102.

The predetermined cut-off rate

In most firms, there exists a need for a cut-off rate which is predetermined by management before projects and the means of financing them are evaluated. This need arises for two reasons. Firstly, the demand schedule for capital, assumed to be known in figure 2, cannot be accurately forecast.

> It should by now be readily apparent that there is a need for some minimum standard or cut-off rate of return in using discounted cash flow evaluations . . . Not all of the investment opportunities which will be available in a forthcoming period are known in advance. Rather, many opportunities come about unexpectedly, and one at a time. Thus, the question is often limited to rejection or acceptance of one project at a time.[66]

Secondly, the cost of funds curve, also assumed to be known in figure 2, defies accurate forecasting. A predetermined cut-off rate is essential if projects are to be correctly evaluated in these circumstances. The value of this predetermined rate should be the same as the cut-off rate determined when all projects and funds available are known in

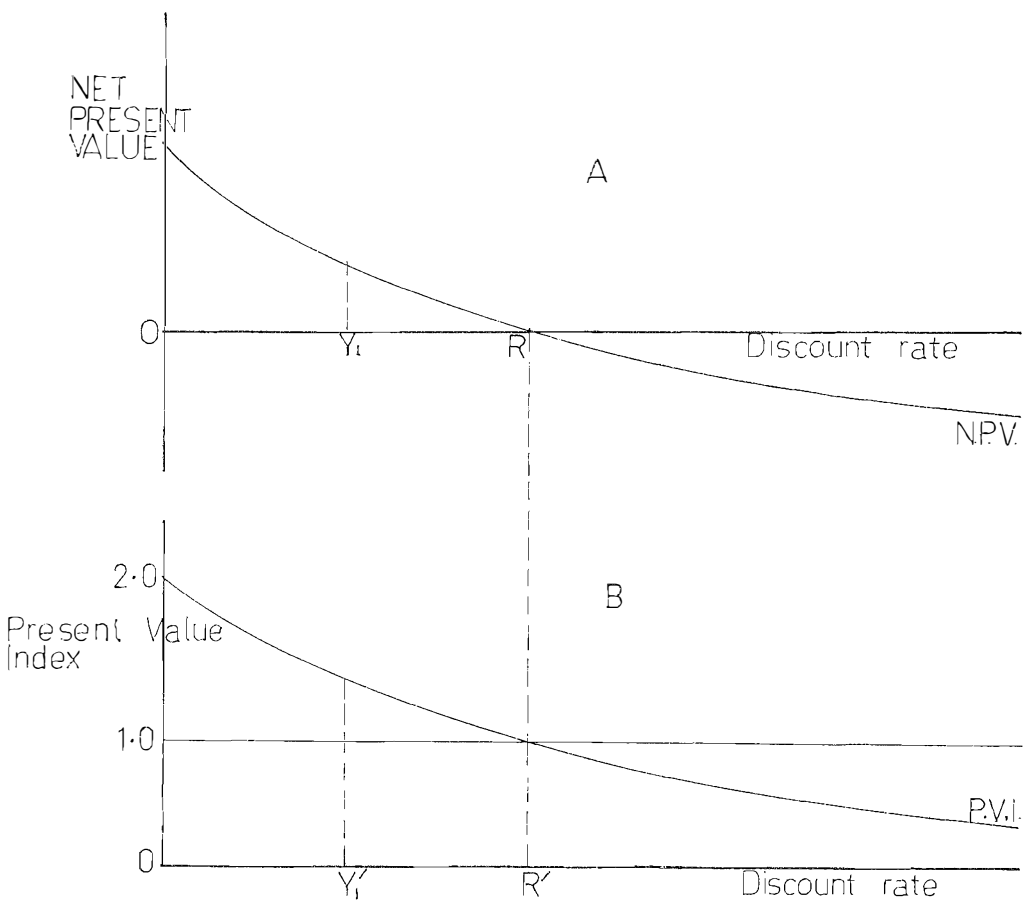

Fig. 3.—Relationship between the cut-off rate and the internal rate of return, net present value and present value index

66 Edwin A. Bowen," Problem Areas in the Use of Discounted Cash Flow for Investment Evaluations", *NAA Bulletin* 44 (August 1963): 18.

advance. However, "in practice the exact point where it occurs is very hazy and difficult to determine".[67] Approximations often have to be used to arrive at practical measures of this cut-off rate.

"Conventional analysis" of cut-off rates under capital rationing

In a common treatment[68] of the cut-off rate under conditions of capital rationing, to be referred to here as the "conventional analysis", a diagram similar to figure 4 is drawn. Competing proposals are ranked in order of descending internal rates of return as shown by the DD curve. The SS curve shows the increasing cost of funds necessary to finance these proposals. Therefore, this diagram is similar to figure 2, shown previously.[69] Where funds are unlimited, the firm could profitably engage in projects up to the point X_1 and the appropriate cut-off rate would be OY_1. However, a funds restriction is imposed and is shown on the diagram by a vertical line X_2L, cutting the DD curve at L. This restriction only permits OX_2 to be spent on capital projects, and it is evident that projects X_2X_1, although profitable, cannot be undertaken. To utilize these available funds in the most profitable manner, a firm should accept proposals along the DD curve up to the point L. The appropriate cut-off rate under this "conventional analysis" then becomes the internal rate of return on the project just rejected because of the funds constraint. This is represented by OY_2 on the y-axis.

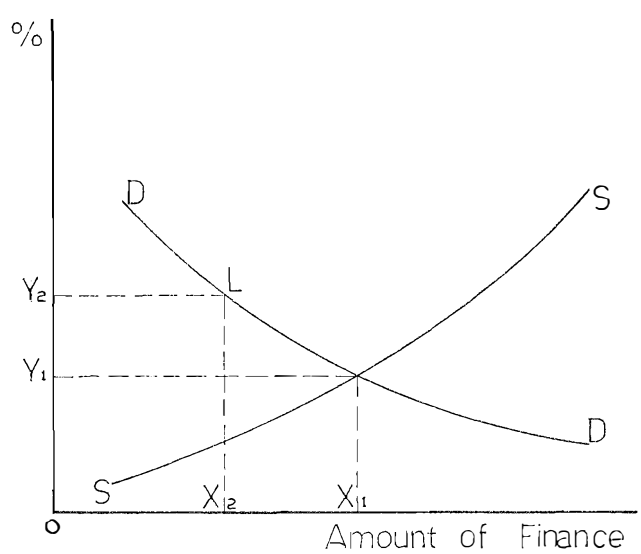

Fig. 4.—"Conventional analysis" of cut-off rates under capital rationing

67 *Ibid.*

68 See Meredith, *Capital Rationing*, pp. 90-97, Merrett and Sykes, *The Finance and Analysis of Capital Projects*, pp. 139-42, and Weston and Brigham, *Managerial Finance*, pp. 177-78.

69 See above, p. 102.

Criticisms of "conventional analysis"

Despite the compelling simplicity of the "conventional analysis", there are three factors which may cause the cut-off rate it determines to fail in the role of a universal cut-off rate under capital rationing.

Inadequacies of the internal rate of return as a ranking criterion

Although the internal rate of return is beset by many difficulties,[70] those of most interest under capital rationing are:

1. The possibility that a proposal has two or more internal rates of return. This can be caused by net cash outflows occurring in a proposal's pattern of cash flows during various periods after its acceptance. Where this occurs, it is not immediately apparent how this project should be ranked.

2. Projects ranked in the demand curve may have varying useful lives. To rank two projects with differing lives properly, some assumption must be made regarding the reinvestment of cash flows from the shorter project over the difference in their lives.[71] This is not handled effectively by the internal rate of return.

3. Use of the internal rate of return to rank proposals assumes that the standard of comparison, the cut-off rate, does not fluctuate over the life of the proposal. It will be demonstrated in section IV that the presence of capital rationing is quite likely to cause future fluctuations in the cut-off rate which consequently the internal rate of return cannot handle.

Implicit assumption regarding the volume of available funds

The "conventional analysis" of capital rationing assumes that the firm has predetermined those volumes of available investment funds and funds for dividend and non-investment payments which will maximize the appropriate goal of the firm. It will be demonstrated in this section that the goal of the firm plays an important part in determining the volume of funds available for investment, and in determining the cut-off rate.

Multi-period investments and the frequency of capital rationing

When analysing investment proposals with useful lives of several years, or "multi-period" investments, the "conventional analysis" takes no account of the frequency of *capital rationing*. Fund restrictions may be imposed on the firm in the current period only, or over several periods, and it would be useful to know how possible alterations in this frequency of rationing affect the cut-off rate. The profitability of an investment proposal having a long useful life could depend on the value of this cut-off rate. If, for example, it fluctuates in future periods when capital rationing is imposed, the net present value or present value index of the proposal will be affected because this future cut-off rate is used to discount future cash flows to their present values.

A substitute for the "conventional analysis"

To examine how these foregoing factors may be taken into account when analysing projects under capital rationing, it is proposed to develop an argument in two parts. The first deals with "two-period" investments. These are proposals which require an investment in period one, and are completely disinvested in period two. Consideration of this kind of investment allows a concentration on the second

70 Elaboration of all the alleged inadequacies of the internal rate of return is outside the scope of this study. However, interested readers may refer to Van Horne, *Financial Management and Policy*, pp. 36-41, and J[ack] Hirshleifer, "On the Theory of Optimal Investment Decision", *Journal of Political Economy* 66 (August 1958): 345-52.

71 This was first pointed out in Solomon, "The Arithmetic of Capital Budgeting Decisions".

inadequacy of the "conventional analysis", mentioned above.[72] This was the possibility that different goals of the firm could cause differing volumes of investment funds to be made available and different cut-off rates to apply. A two-period project automatically avoids the possibility of differing frequencies of capital rationing, since capital is rationed in one period only.

Many of the inadequacies of the internal rate of return are overcome also. Specifically, as each project consists of an initial outlay and one cash return, there is no possibility of multiple internal rates of return; and as all projects automatically have the same useful lives, there is no problem with equating useful lives to obtain meaningful internal rates of return.

It is assumed that all projects competing for available funds are divisible so it would be possible to draw a smooth demand curve for capital similar to the DD curve in figure 2.[73] This assumption will be maintained in the rest of this section and in section IV, since it greatly simplifies the argument to be presented.

The second stage of the argument, to be outlined in section IV, deals with "multi-period" investments. This allows discussion of the frequency of capital rationing and its effect on the cut-off rate.

Two-period investments under capital rationing

The goal of the firm

Where funds have been limited either by the capital market or by management, the appropriate investment objective has been stated in the following words:

> In the presence of capital rationing one is no longer concerned with the selection of an ideal set of investments whose acquisition might well require resources beyond those available to the firm. Rather, one must determine the best course which can be followed with the limited funds on hand.[74]

However, this does not answer the question "The best course for whom?" If it is the best course to adopt for the benefit of the *firm*, then the objective becomes determining that set of investment projects which maximizes profits of the firm subject to the funds constraint.

On the other hand, if the best course to adopt is for the benefit of the ordinary shareholders, then the objective may not be to adopt a set of proposals which maximizes the profits of the firm. It should be to maximize the wealth of the ordinary shareholders. This consists of a subtle mix of increases in the market value of ordinary shares, and increases in dividend payments, without diluting ordinary shareholders' equity.[75]

It is assumed in this study that the market price of a share depends directly on the future dividends to be paid on it. This follows John B. Williams' theory of investment value which states that an ordinary share is worth the sum of all expected future dividends discounted to the present.[76] Therefore, the wealth of the ordinary shareholder, in theory at least, depends on the generation of current and future dividends.

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72 See above, p.106.

73 See above, p. 102.

74 William J. Baumol, *Economic Theory and Operations Analysis*, 2nd ed. (Englewood Cliffs, New Jersey: Prentice-Hall, 1965), p. 448.

75 For a further discussion of this goal, see Van Horne, *Financial Management and Policy*, pp. 7-8, and James T. S. Porterfield, *Investment Decisions and Capital Costs* (Englewood Cliffs, New Jersey: Prentice-Hall, 1965), pp. 11-17.

76 See John B. Williams, *The Theory of Investment Value* (Cambridge: Harvard University Press, 1938).

Ploughing all the firm's resources into capital projects implies that ordinary shareholders have an absolute preference for future dividends and are prepared to forego all current dividends to get them. On the other hand, paying out all resources as current dividends implies that these shareholders prefer current dividends to future dividends and are prepared even to liquidate the firm to get them. In many companies, however, neither situation applies. Instead, ordinary shareholders desire current dividends but also want to see a proportion of the firm's resources ploughed back into investment projects, so that future dividend prospects are enhanced.

> Although investors have prized dividends, they have never expected companies to pay out the full amount of their earnings. It is considered sound corporate policy, and thus in the interests of the shareholders, to retain an appreciable part of an average year's earnings for various protective and constructive purposes.[77]

At this point it shall be assumed that all ordinary shareholders in a firm have the same expectations and desires regarding the stability of current and future dividends from their firm. They are in the same income stratum in society, are possessed of the same personality characteristics and are faced by the same set of consumption opportunities to which they can apply their dividends. Also, these dividend payments are their only source of income. In short, these ordinary shareholders belong to one homogeneous group in society.[78]

Bearing this assumption in mind, it is further postulated that the question of current or future dividend payments to these shareholders can be resolved by reference to a common schedule of time preference rates.

> Thus, it is assumed that the [ordinary shareholders] have a schedule of time-preference rates by means of which consumption opportunities are evaluated. One may conceive of this schedule as serving two purposes. First, it provides a means of comparing current consumption opportunities with each other and thus a means of allocating funds among specific increments of consumption. Second, it provides a means by which the [ordinary shareholders] may choose between consumption and investment opportunities, that is, between immediate and delayed consumption. [Their] objective in making . . . financial decisions should be to consume, invest, and raise funds in such a way that [their] consumption over time will yield [them] the greatest wealth.[79]

Since dividends are the only source of income to these ordinary shareholders, this schedule of time preference rates can be mapped as a set of indifference curves relating current dividends to future dividends. This is shown in figure 5.

Current dividends are drawn on the x-axis and future dividends are drawn on the y-axis. For a firm concerned only with two period investments, the y-axis would become period one dividends, but this development is explored later.[80] The curves u_1, u_2, and u_3 are just three of a family of curves that could be drawn on the diagram, but are not for the sake of simplicity. Each curve is the locus of points, representing combinations of current and future dividends between which the ordinary shareholders are indifferent. Thus, ordinary shareholders would be indifferent if they had to choose between either a combination of X_1 current dividends and Y_1 future dividends, or a combination of X_2 and Y_2 of the same dividend opportunities.

If it is the aim of ordinary shareholders to maximize their wealth they would prefer to be on indifference curve u_3 rather than u_2 or u_1, since all combinations on

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77 Benjamin Graham, David L. Dodd, and Sidney Cottle, *Security Analysis: Principles and Technique*, 4th ed. (New York: McGraw-Hill Book Co., 1962), p. 482.

78 This rather bold assumption will be relaxed later. See below, p. 114.

79 Porterfield, *Investment Decisions and Capital Costs*, p. 9.

80 See below, p.110.

u_3 represent more dividends than any point on u_2 or u_1. In striving to achieve the goal of wealth maximization for its ordinary shareholders, a firm should attain that combination of current and future dividends that puts these shareholders on the highest indifference curve possible, given the level of current resources available to the firm. By relating the allocation of funds for investment proposals to the question of current or future dividend payments, a firm with rationed funds may achieve the goal of maximization of ordinary shareholders' wealth.

Even where the goal of the firm is *corporate* profit maximization, the question of dividend payments arises where only limited funds are available, because a choice must be made between dividends and investment projects. A firm in this situation would try to pay as small a dividend as possible in the hope of utilizing all profitable investment projects.

It could be concluded that the goal of the firm plays a vital part in capital expenditure analysis when funds are limited. This goal determines the appropriate amount of dividend payments made, and these in turn determine the volume of retained funds available for capital expenditures.

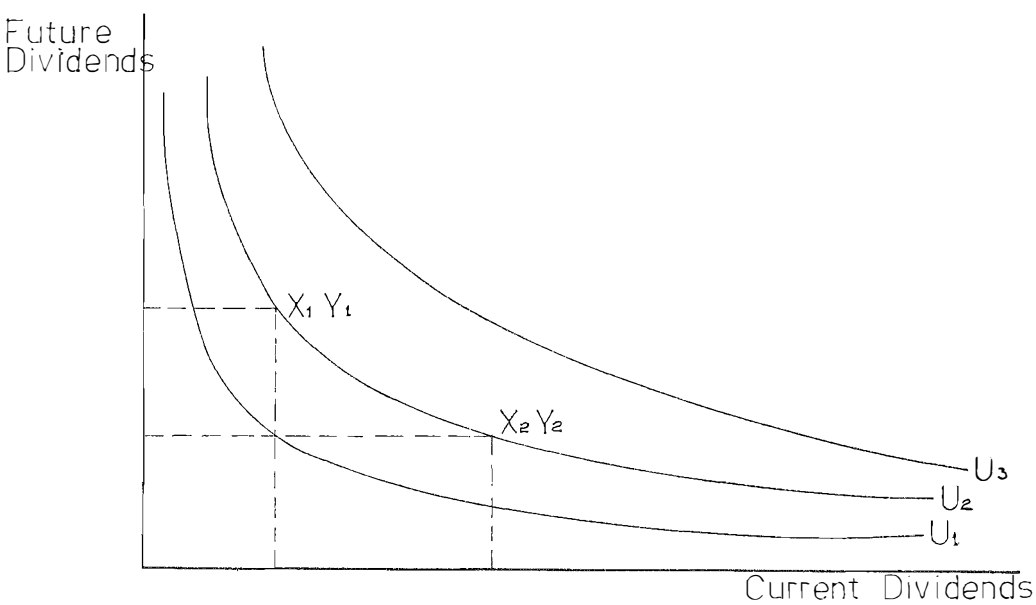

Fig. 5 — Ordinary shareholders' indifference map relating current and future dividends

A fallacy inherent in the "conventional analysis"

In the "conventional analysis" of cut-off rates under capital rationing, there is an implicit assumption that the "correct" amount of dividends has been paid before funds available for capital expenditures are determined. Although the goal that is being aimed at by advocates of the "conventional analysis" is sometimes stated,[81]

81 For example, the goal of shareholders' wealth maximization is stated by Merrett and Sykes, *The Finance and Analysis of Capital Projects*, p. 142. However, they fail to relate the dividend question to their analysis.

they fail to incorporate in their analyses the combination of dividend payments and investments necessary to maximize their particular goal.

The following analysis[82] incorporates the question of differing goals and the related question of dividend payments.

Basic data for the two-period analysis

Consider a firm that has available in period zero a fixed volume of funds which are to be used for dividend payments and capital investments. The limit on available funds may be either internally or externally imposed.

This firm has a number of independent two-period investment projects available which can be arranged in order of profitability. Profitability is defined as the internal rate of return on each proposal. Where an initial outlay K_0 in period zero is required for a return of K_1 in period one, the internal rate of return is given by r in the equation

$$K_0 = \frac{K_1}{(1 + r)}$$

Solving for r, this equation becomes

$$r = \frac{K_1}{K_0} - 1$$

All available internal two-period investments can be arranged in descending order of profitability to form what shall be called the productive opportunity curve for the firm. "It is the locus of points attainable to [the firm] as [it] sacrifices more and more of K_0 by productive investments yielding K_1 in return."[83] This productive opportunity curve is presented in figure 6 by the curve XYZB.

The x-axis of figure 6 shows resources in period zero and OX is the limit of available funds. Income generated in period one by undertaking two-period investments is shown on the y-axis.

The curve XYZB shows investment opportunities open to the firm with OX funds available in period zero. For instance, by investing all OX in two-period projects, the firm will accumulate OB funds in period one.

Alternatively, by investing XE funds in projects, OE funds are available for period zero dividend payments, and OH funds are generated in period one. By committing XE of current funds to investments, the firm would accept all the projects represented by the segment XYF of the productive opportunity curve XYZB. The internal rate of return of the project at any point on this segment is given by the slope of the productive opportunity curve at that point. Thus, by investing along this curve from X to F, the firm is accepting projects of strictly decreasing profitability since the curve is steepest at X and flattens out in proceeding through the point Y to F. By drawing the productive opportunity curve as a smooth curve, recognition is given to the assumption made earlier, that all proposals under consideration are divisible.[84]

The indifference curves u_1, u_2, and u_3 are part of the assumed known schedule of time preference rates for current dividends as opposed to future dividends. In the case being considered here, future dividends would be period one dividends. Once the two-period proposals have been disinvested, their receipts are assumed to be disbursed to ordinary shareholders. In this way, assumptions about the future reinvestment of funds by the firm can be avoided. Aside from this qualification, however, these indifference curves are the same as those shown in figure 5.[85]

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82 This analysis is based on Hirshleifer, "Optimal Investment Decision".

83 *Ibid.*, 332.

84 See above, p. 80.

85 See above, p. 109.

Apart from the array of internal investment opportunities displayed by the curve XYZB in figure 6, the firm has other two-period investment opportunities open to it. These consist of investments in the best opportunity external to the firm. For instance, this opportunity could be an investment in government securities. By investing all period zero resources in this "next-best" opportunity, the firm would obtain OA income in period one. The slope of line AX is the rate of return available from this investment. Similarly, OC is the period one return available from an investment, in this opportunity, of period zero resources well in excess of OX and not shown in figure 6. The amount of these period zero resources is given by the imagined intercept of the line CZ with the x-axis. The slope of CZ equals the slope of AX because both investments have the same rate of return.[86]

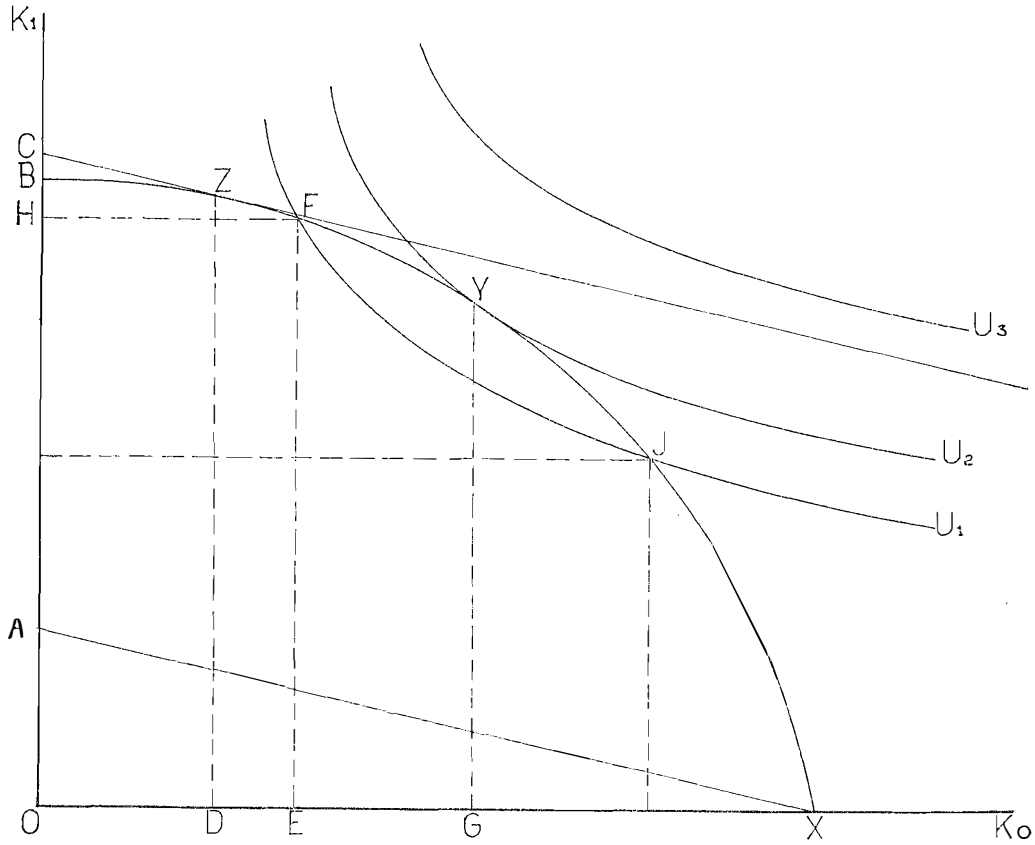

Fig 6. — Two-period investments under capital rationing

The slope of these lines represents the cost of capital to the firm. There are no explicit costs of capital since the firm has no access to the capital market and so cannot raise funds by issuing shares or by borrowing. This cost of capital shown is an opportunity cost of using funds already in the firm. It is assumed to be constant during periods zero and one, an assumption represented by drawing AX and CZ as straight lines.

86 There would be a family of lines parallel to AX, showing the outcomes of investing various amounts of period zero funds in this "next-best" opportunity.

With these facts about figure 6 discussed, an outline will be given of the correct cut-off rates under both the goals of corporate profit maximization and maximization of the wealth of the ordinary shareholders.

The cut-off rate under corporate profit maximization

If the firm could relegate dividend payments to a secondary place, corporate profits would be maximized by selecting projects in figure 6, from X along the productive opportunity curve to Z, the point of tangency between this curve and the cost of capital line CZ. Here the slope of XYZB becomes equal to the cost of capital, implying that the internal rate of return of project Z equals the opportunity cost of capital. Investments in excess of Z along the segment ZB would have internal rates of return less than the cost of capital and are unprofitable. If the firm could escape all dividend payments, OD not invested internally, would be invested in the "next-best" opportunity along CZ, and OC funds would be the total generated in period one.

However, management of the firm may be obliged to pay dividends. The shareholders may threaten to change the management team unless dividends are paid, or management may fear a take-over bid for the firm if they pay no dividends. Suppose that the minimum dividend that must be paid in period zero is OE. This leaves XE available for investment. Since all projects from X to Z along the productive opportunity curve have been deemed profitable, the firm should invest all remaining resources XE in them. Thus, proposals X to F are undertaken.

The appropriate cut-off rate in this case is the slope of the productive opportunity curve at the point F. It represents the internal rate of return of the project just rejected. This result would be similar to the cut-off rate determined under the "conventional analysis" if the only projects being considered in that analysis were two-period investments.[87]

The predetermined cut-off rate under corporate profit maximization

In the foregoing analysis, it has been assumed that all projects competing for available funds are known in advance. However, as shown previously,[88] where advance knowledge of all projects is not possible, a predetermined cut-off rate is required. To obtain this rate for two-period projects, it may be necessary to look at the marginal internal rate of return on proposals just rejected in the past. However, for this to be a valid step, it must be assumed that the volume of funds available and the profitability of investments in prior years resembled those in the current year. The objective is to approximate the current year situation depicted in figure 6, and defined by the volume of available funds OX as well as the profitability of available projects. These two variables determine the shape and position of the productive opportunity curve XYZB. If the above conditions were present in previous years, the investment situation then in existence could be depicted in a diagram similar to figure 6. Theoretically, the predetermined cut-off rate would be the internal rate of return of the project just rejected in prior years. However, where advance knowledge of the full array of available projects was not available in the past, the correct predetermined cut-off rate is the internal rate of return of the proposal just *accepted* in these years. Since, by assumption, all projects under consideration are divisible, the project just accepted in the past would be only infinitesimally more profitable than the one just rejected. Such a small difference is immaterial for practical purposes.

Where discounted cash flow methods were not used previously, no such marginal internal rate of return is available immediately. Nevertheless, this required rate may be obtained in the following manner:

87 See above, p. 105.
88 See above, p. 104.

The internal rate of return of any two-period project,[89]

$$r = \frac{K_1}{K_0} - 1, \text{ where } K_1 = \text{income in period 1}, \quad K_0 = \text{outlay in period zero}$$

$$= \frac{K_1 - K_0}{K_0}$$

However, $K_1 - Ko/Ko$ is the accounting rate of return for this project. Therefore, the required marginal internal rate of return on the project just accepted in prior years is obtained by using the accounting rate of return on this prior period project.

The cut-off rate under maximization of the ordinary shareholders' wealth

Where the goal of the firm becomes maximizing the wealth of its ordinary shareholders, the cut-off rates computed in the previous section are not correct. Ordinary shareholders' wealth is maximized by selecting that combination of dividends and investments in period zero which puts these shareholders on the highest possible indifference curve shown in figure 6. The highest such curve attainable is u_2 which is a tangent to the productive opportunity curve XYZB at the point Y. Projects are accepted up to the point Y and the quantity of period zero funds invested is XG. The remainder GO is distributed as current dividends. Ordinary shareholders obtain YG as dividends in period one.

That investment in projects up to the point Y results in maximum wealth accruing to the ordinary shareholders can be seen by considering any other point on the productive opportunity curve XYZB, such as the point J. The highest indifference curve attained there is u_1, which is less than indifference curve u_2. Since u_2 is tangent to the XYZB curve it is the highest indifference curve attainable.

The appropriate cut-off rate in this case is the internal rate of return of the project just rejected, and is represented by the slope of the XYZB curve at the point Y.

The predetermined cut-off rate under maximization of ordinary shareholders' wealth

Although the foregoing theoretical result may seem straightforward, actual derivation of the slope of the productive opportunity curve at Y is extremely difficult. There are two reasons why this is so.

Interdependency between the cut-off rate and the shape of the productive opportunity curve

A fundamental weakness exists in the previous theory which prompts the search for a substitute cut-off rate. It simply amounts to saying that the firm does not know the shape of its productive opportunity curve in advance. Therefore some predetermined cut-off rate is required.[90] However, there is more to the matter than this. Hirshleifer has described the situation in the following terms:

> The discount rate to be used for calculating present values or as a standard of comparison against the internal rate of project increments is the rate given by the slope of the . . . tangency (the marginal Internal Rate of Return); with this rate, the rules give the correct answer. But this rate cannot be discovered until the solution is attained and so is of no assistance in reaching the solution.[91]

Baumol and Quandt also explain this weakness:

> The trouble simply is that in the circumstances under consideration we cannot determine a discount rate until we can calculate a marginal profit opportunity

89 See above, p. 110.

90 This weakness is evident also where maximization of corporate profits is the assumed goal of the firm. It was dismissed previously by saying that the shape of the productive opportunity curve is not known in advance and a predetermined cut-off rate is needed.

91 Hirshleifer, "Optimal Investment Decision", p. 334.

cost of postponement; but we cannot calculate profit until we have obtained our discount rate.[92]

The underlying cause is the interdependency between the slope of the productive opportunity curve and the cut-off rate. For instance, knowledge that this rate is likely to be quite high since funds are *very* limited may influence management to put extra efforts into project generation so that the resulting array of proposals have very high profitability. On the other hand, if it is thought that the cut-off rate will be relatively low, there may be an air of indifference to the task of generating projects of high profitability. This interdependency between the cut-off rate and the shape of the productive opportunity curve causes the determination of one in isolation of the other to be a difficult task.[93]

Rather than wrestle with this problem, Baumol and Quandt have suggested an alternative approach. Although the accuracy of this alternative is not absolute, it is the best available in the light of the difficulties involved in estimating the marginal internal rate of return required to select projects as a means of maximizing ordinary shareholders' wealth. The Baumol and Quandt solution has been summarized by Van Horne.

> They [Baumol and Quandt] suggest the use of a subjective discount rate, determined by the utility preferences of the entrepreneur . . .[94]

Management of the firm determines the appropriate cut-off rate under the Baumol and Quandt approach. It would be based on an assessment of the profitability of marginal projects in prior years tempered by subjective factors. These subjective factors include the attitude of management towards the risk inherent in the proposals submitted, the state of the economy and the future prospects of the firm. A conservative management might require a higher cut-off rate than in previous years because the flock of projects currently submitted for appraisal is more risky than projects in the past. Optimism on the part of management might cause them to lower the cut-off rate in the hope of reaping the extra profits they think are available "if only investments are made now".

The impossibility of obtaining a unique set of ordinary shareholders' indifference curves

There is another weakness underlying the two-period analysis. Where ordinary shareholders' wealth is to be maximized, a common schedule of time preference rates for these shareholders was used in deriving the correct cut-off rate. This schedule was represented by the indifference curves u_1, u_2 and u_3 in figure 6, and to obtain them, complete homogeneity of all shareholders was assumed.[95] However, such an assumption is unrealistic. Shareholders typically belong to many heterogeneous groups with differing levels and sources of incomes, often contradictory cultural backgrounds, and conflicting desires and aspirations about their firm. With such diversity, no unique set of indifference curves is possible.

To overcome this difficulty, the subjective discount rate of management is again proposed. Its use will result in the maximization of ordinary shareholders' wealth where it is assumed that management has the best interests of these shareholders at heart.

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92 William J. Baumol and Richard E. Quandt, "Investment and Discount Rates Under Capital Rationing—A Programming Approach", *Economic Journal* 75 (June 1965): 325.

93 This argument is based on a line of reasoning used to explain the interdependency of the market rate of interest and the shape of the productive opportunity curve where no capital rationing exists. For further information see Irving Fisher, *The Theory of Interest: As Determined by Impatience to Spend Income and Opportunity to Invest It* (New York: Macmillan Company, 1930), p. 278, and Armen A. Alchian, "The Rate of Interest, Fisher's Rate of Return over Costs and Keynes' Internal Rate of Return", *American Economic Review* 45 (December 1955): 942.

94 Van Horne, *Financial Management and Policy*, n. 14, p. 44.

95 See above, p. 108.

Summary and conclusions

The discussion of the cut-off rate under capital rationing for a two-period investment is summarized in table 9.

TABLE 9

Summary of cut-off rates for two-period investments

Goal of firm	Theoretical cut-off rate	Predetermined cut-off rate
Corporate profit maximization	Marginal internal rate of return of project just rejected	Accounting rate of return of project just accepted in the past
Maximizing ordinary shareholders' wealth	Internal rate of return of project at the point of tangency between the indifference curves of shareholders and the productive opportunity curve	Management's subjective discount rate

The two-period analysis presented in this section implies that projects are selected on the basis of their internal rates of return, since the productive opportunity curve was devised by ranking proposals in descending order of their internal rates of return. Selection was made of all proposals whose internal rates of return exceeded the cut-off rate, by choosing proposals along the productive opportunity curve in figure 6, until the cut-off point was reached.

However, this analysis should not be taken as a negation of the use of criteria such as the net present value or present value index. By using as discount rates the cut-off rates appropriate under the goals of corporate profit maximization or ordinary shareholders' wealth maximization, both the net present value and present value index criteria will lead to selection of the sets of investments which achieve these goals.

The analysis presented in this section was concerned with two-period investments and so would be of limited use to most firms. In practice, most investments that are undertaken last many years, during which the firm could experience capital rationing in periods of long or short duration. To cater for these cases, multi-period investments under capital rationing will be considered in section IV.

IV. THE CUT-OFF RATE FOR MULTI-PERIOD INVESTMENTS UNDER CAPITAL RATIONING: THEORY AND PRACTICE

Introduction

Multi-period investments, or those investments with useful lives in excess of two-periods, raise special problems when they are to be evaluated under conditions of capital rationing. Perhaps the most important of these problems is the frequency of capital rationing and its effect on project acceptability. Sometimes capital restrictions exist only in the current period, and thereafter all rationing is lifted, and every profitable investment may be utilized. This situation will be called "single-period rationing". On the other hand, funds retrictions may be enforced over several capital budgeting periods, perhaps even as long as the lives of all proposals being considered currently. Such a situation will be called "multi-period rationing". As each form of capital rationing involves different sets of cut-off rates, multi-period investments will be analysed in two sections here: the first dealing with "single-period rationing", and the second dealing with "multi-period rationing".

Since multi-period investments are likely to be the most common variety of capital expenditure found in practice, it would be interesting to discover the answers to

these questions: Are the cut-off rates devised in this section being utilized by businessmen? If they are not, what procedures do businessmen use to adjust for capital rationing? To answer these questions, some empirical evidence will be presented from the survey of listed Australian public companies performed by the writer. This evidence gives a guide to the current state of practice in this area.

Single-period rationing

Where the firm imposes a restriction on the funds available in the current period, and none thereafter, the appropriate cut-off rate for evaluating future projects returns to the "normal" rate applicable when funds are unlimited. If the goal of the firm is maximization of ordinary shareholders' wealth, this "normal" cut-off rate is the firm's cost of capital.

> We first referred to cost of capital as a "hurdle rate" or "cut-off rate" to be used when evaluating proposed capital expenditures by the rate-of-return approach. Thus if a project offers a rate of return greater than the cost of capital, it is worthwhile from a monetary standpoint. If the cost of capital exceeds the expected rate of return, the residual owners would be worse off financially if the project were undertaken. Alternatively, cost of capital serves as the discount rate in the net present value approach.[96]

In addition, if the goal is corporate profit maximization, the appropriate cut-off rate is again the cost of capital. A firm should not invest in proposals with profitabilities less than the costs of funds necessary to finance them.

However, actual computations of the cost of capital under each of these goals are likely to be different, simply because each goal is different.

> Determining cost of capital is an extremely complex problem, the dimensions of which can only be suggested here. Since capital budgeting, by definition, involves choosing future courses of action, it is a *future* cost of capital which is needed. But some value judgments concerning management's conception of the firm's principal objective(s) must be made initially to establish a viewpoint from which this cost can be measured. One might, for example, measure cost of capital differently if management's primary goal were viewed as maximization of corporate net worth than if maximization of returns to shareholders were considered management's prime objective. (Especially is such a divergence in measures possible with respect to the cost of equity capital.)[97]

The theoretical computation of costs of capital under each of these goals will not be discussed further,[98] but it should be remembered that when reference is made subsequently to the costs of capital under these goals, the two costs of capital are unlikely to be the same.

If the firm can compute its cost of capital in all future periods, then the only problem remaining is to determine the appropriate cut-off rate for the current period. From the previous discussion of two-period investments,[99] it is likely that this rate will differ from the cost of capital.

Derivation of the current period cut-off rate

Whilst the nature of current investments plays a part in determining the volume of internally generated funds available in the future, it can be safely assumed that where only single-period capital rationing is imposed, this volume of funds does not

96 Johnson, *Financial Management*, p. 275. The rate of return referred to by Johnson is the internal rate of return of the project.

97 Victor H. Brown, "Rate of Return: Some Comments on its Applicability in Capital Budgeting", *Accounting Review* 36 (January 1961): 55.

98 However, interested readers could refer to Van Horne, *Financial Management and Policy*, pp. 109-139.

99 See above, p. 115.

restrict future dividend payments and future investment decisions. If the pattern of future cash flows from investments is such that the firm expects to find itself short of internal resources in a future period, then it may obtain additional funds from the capital market to supplement those internally generated.[100] As a result, dividend payments may be maintained and the firm can engage in all profitable capital expenditure proposals in that period.

Because of this ready access to the capital market in the future, there is no need to adjust the capital expenditure analysis of current investment projects to ensure that those selected now will generate large cash benefits in that future period to increase the internal funds of the firm. Therefore if two investment projects have the same useful lives, but one has an irregular pattern of cash flows and the other has a constant annual cash flow, they will be equally desirable, provided they both have the same internal rate of return.

If the firm had available a selection of independent, divisible investments all with infinite lives and irregular patterns of cash inflows, it would be possible to stipulate for each proposal, a perpetuity[101] with an identical initial outlay and the same internal rate of return. The firm would be indifferent between this perpetuity and the original proposal.

By considering all infinite proposals in terms of their perpetuity equivalents, it is possible to look upon the activities of the firm as occurring in two periods. Firstly, there is the current period, or period zero, in which the competing investments are analysed and those selected are undertaken. Secondly, all periods after the current period may be regarded as one long "future" period, stretching from period one to infinity. Investments in period zero then return a constant amount at regular intervals forever during this future period. Therefore, this assumes that everything concerning the welfare of the firm continues unchanged forever after period zero. Looked at in this light, all projects of infinite life can be treated as two-period investments and the cut-off rate in period zero is derived in a similar fashion to the cut-off rate for two-period investments derived in section III.

However, in reality most projects have definitely limited lives[102] and there is no justification for finding perpetuities with identical internal rates of return to these finite proposals. The life of the firm itself is open to question beyond more than several decades, since the planning activities of the firm become very inaccurate for periods far into the future, and the cash flows from projects with very long lives become

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100 It is assumed that the cost of capital in future years remains relatively stable.

101 A perpetuity is defined as a project with an initial outlay C_0, and a constant annual return S, forever. The internal rate of return for this perpetuity is the rate r in the equation:

$$C_0 = \frac{S}{1+r} + \frac{S}{(1+r)^2} + \ldots + \frac{S}{(1+r)^n} \qquad \text{(1)}$$

$$(1+r)C_0 = S + \frac{S}{1+r} + \ldots + \frac{S}{(1+r)^{n-1}} \qquad \text{(2)}$$

Subtracting (1) from (2)

$$(1+r)C_0 - C_0 = S - \frac{S}{(1+r)^n}$$

$$r = \frac{S}{C_0} - \frac{S}{C_0} \cdot \frac{1}{(1+r)^n}$$

Where the life of the project is infinite, $\frac{1}{(1+r)^n} = \text{zero}$

Therefore, $r = \frac{S}{C_0}$

102 A possible exception would be an investment in a block of land.

subjected to high degrees of uncertainty. Nevertheless, where the firm does plan ahead for a reasonable period into the future, say fifty years, it is possible to treat all projects as fifty-year investments generating constant annual cash flows, as will now be shown.

Suppose that all the proposals submitted to the firm have lives ranging from three years up to fifty years. For meaningful analysis, it is necessary to compare these proposals over the economic life of the longest project; that is, over fifty years. Therefore, some assumption must be made regarding the reinvestment of cash flows from the short projects in the interim period. It will be assumed that necessary information about these reinvested cash flows is available.[103] This means that each proposal can be represented as a fifty-year project whether its original life is fifty years or not. For instance, a proposal of ten years' economic life could be represented as follows:

Expected cash flows $S_1, S_2, \ldots, S_{10}, S_{11}, S_{12}, \ldots, S_{50}$, where
$S_1, S_2, \ldots, S_{10}$ are net cash flows expected to be generated by the proposal; and
$S_{11}, S_{12}, \ldots, S_{50}$ are net cash flows expected to be generated by funds from this proposal, reinvested in the firm.

If the firm's cost of capital during all future periods is known, then this irregular series of cash flows may be converted into a series of constant annual cash flows over the fifty-year period. This is achieved by computing the period one present value of all future cash flows from the proposal, and then converting this into a constant annual sum.

$$\text{Present value in period one} = S_1 + \frac{S_2}{1+i} + \ldots + \frac{S_{50}}{(1+i)^{50}}, \text{ where}$$

$S_1, S_2, \ldots, S_{50}$ are the cash flows as before,
and i is the cost of capital.

$$\text{Constant annual sum}[104] = \frac{\text{Present value in period one}}{1 + \frac{1}{1+i} + \frac{1}{(1+i)^2} + \ldots + \frac{1}{(1+i)^{50}}} = S^*$$

Bearing this in mind, the firm's activities can then be regarded as being carried out in two periods. The first period is period zero in which single-period rationing exists, and in which the available fifty-year investment projects are evaluated. The second period is a "future" period, stretching from the close of the first period to the fifty-year planning horizon. Beyond this "future" period it may be assumed for

103 This may be obtained from statistical projections of past earnings figures for the firm, or it may be based on management's budgeted profit figures during these periods.

104 If the initial outlay of this proposal is C_0, its internal rate of return r is given by the equation:

$$C_0 = \frac{S^*}{1+r} + \frac{S^*}{(1+r)^2} + \ldots + \frac{S^*}{(1+r)^{50}}$$

Multiplying both sides by $(1 + r)$

$$(1+r)\,C_0 = S^* + \frac{S^*}{(1+r)} + \ldots + \frac{S^*}{(1+r)^{49}}$$

By subtraction of the previous equation from this one

$$(1+r)\,C_0 - C_0 = S^* - \frac{S^*}{(1+r)^{50}}$$

$$r\,C_0 = S^* - \frac{S^*}{(1+r)^{50}}$$

$$r = \frac{S^*}{C_0} - \frac{S^*}{C_0}\,\frac{1}{(1+r)^{50}}$$

practical purposes that the activities of the enterprise cease. During this "future" period, the welfare of the firm does not change, it has ready access to the capital market at a constant or at most, a moderately rising cost of capital, and investments undertaken in period zero do not affect decisions to pay dividends or to undertake investments in the "future" period.

Period zero projects can be considered in terms of their equivalents, fifty-year projects with constant annual returns. For each project, an outlay is made in period zero, in return for which the firm receives a constant annual sum at regular intervals over the "future" period.

It is possible to use these facts in analysing the cut-off rates in period zero, using an argument similar to that presented for two-period investments in section III. While this argument is deficient to the extent that it does not consider the future of the firm beyond fifty years, it is thought that for the task of deriving period zero cut-off rates, the future of the firm beyond fifty years does not matter very much anyway. A diagram such as figure 7 is used to facilitate the argument.

Funds in period zero are shown on the x-axis. There is a limit OX to these funds, which may be either internally or externally imposed. On the y-axis is shown the equivalent constant annual returns available to the firm from investing successive increments of period zero resources in projects along the productive opportunity curve XABC. This curve shows the internal investment opportunities open to the firm and its slope at any point is given by the ratio S^*/C_0. This ratio approximately equals the internal rate of return available from the project, as can be seen from the following table.

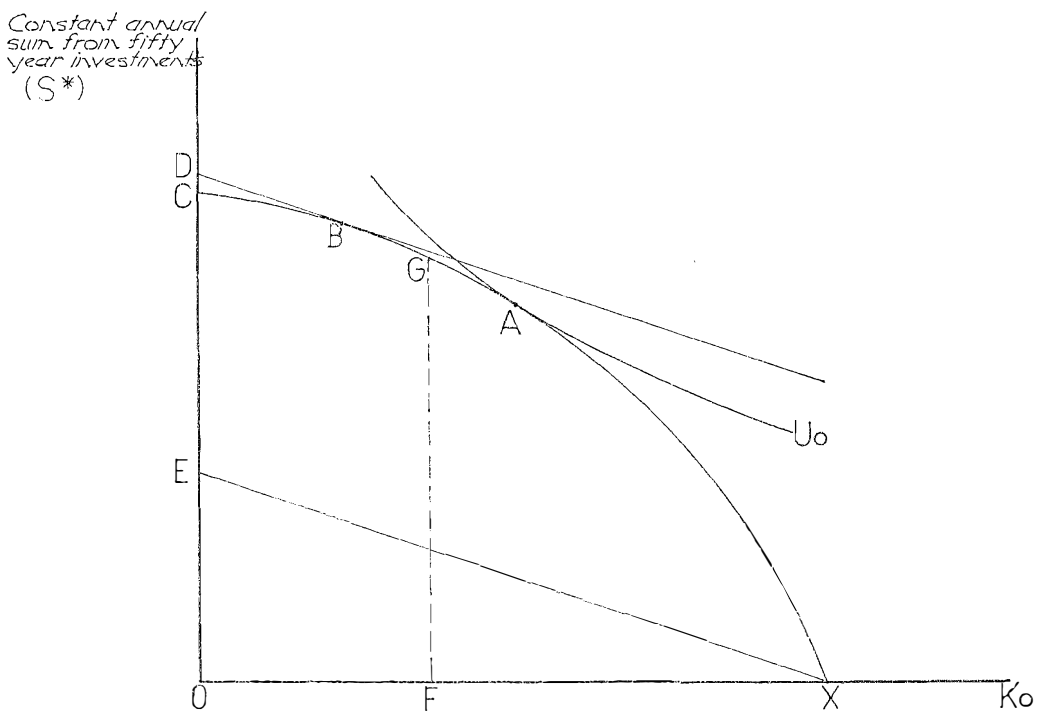

Fig. 7.—Multi-period investments under single-period rationing

TABLE 10

Relationship between the internal rate of return and the ratio S*/Co

Internal rate of return %	S^*/C_0 %
0	2
5	5.43
10	10.1
15	15.05
20	20.05
25	25.05

This table is based on the equation for internal rate of return

$$r = \frac{S^*}{C_0} - \frac{S^*}{C_0}\,\frac{1}{(1+r)^{50}}$$

For internal rates of return in excess of 5%, the ratio S^*/C_0 is a good estimate of the internal rate of return. Therefore, projects on the segment of the curve XA have higher internal rates of return than projects on the segment AG because the slope of the curve at all points on XA exceeds the slope at all points on AG.

It was assumed in section III,[105] that a common schedule of time preference rates for the ordinary shareholders was known and could be drawn as a series of indifference curves. The same assumption is initially made in this section. However, only one of these curves is drawn in figure 7 and this is the curve u_0.

The slope of the line XE represents the cost of capital of the limited funds OX. This cost is the rate of return available by investing period zero resources in the best available opportunity external to the firm. If all period zero funds were invested in this external opportunity, OE funds would be generated in period one.

The line BD has the same slope as XE, and is a tangent to the productive opportunity curve at the point B.

The cut-off rate under corporate profit maximization

Suppose the firm is obliged to distribute OF funds as dividends in period zero because the ordinary shareholders demand this amount or because other factors apply. If the firm had no restrictions on available funds and did not have to pay any dividends it would invest in internal projects along the productive opportunity curve up to the point B where the marginal internal rate of return equals the cost of capital. To engage in proposals along the segment of the curve BC would be detrimental to the firm's profitability, because the internal rates of return on projects in this segment are less than the cost of capital. The optimum investment would be to accept internal projects from X to the point B on XABC, and to invest remaining funds in the "next-best" opportunity represented by the cost of capital line BD. The firm would generate OD funds in period one.

However, since a funds limit XF is imposed, the firm is in a situation of capital rationing. The optimum investment is to accept projects from X to G along the productive opportunity curve. The cut-off point is the internal rate of return of the project just rejected and is represented by the slope of the productive opportunity curve at the point G.

Predetermined cut-off rate under corporate profit maximization

Where the array of investment proposals needed to discover the correct cut-off rate is not known in advance, a predetermined cut-off rate is required to evaluate proposals as they come to hand. If discounted cash flow methods of capital expenditure analysis have been used in the past, this cut-off rate would be the internal rate

105 See above, p. 108.

of return of the project just accepted in a past capital rationing period. Ideally, for this to be the correct rate, the time period over which all proposals were compared in the past has to be identical to the time span covered by current investment projects. In addition, the level of available investment funds in this past capital rationing period must approximate the level of current investment funds, and all projects in the past period should be of comparable profitability to current proposals. These requirements are imposed for the same reasons governing the restrictions on deriving the predetermined cut-off rate for two-period investments.[106]

Where the firm has not used discounted cash flow techniques in the past, but has been subjected to single period rationing in some years, derivation of the appropriate predetermined cut-off rate in the current period is slightly more complicated.

It will be recalled that once the cash flows from any project are compared over fifty years, the assumed life of the longest project currently under consideration, it is possible to convert the series of uneven cash flow into a series of equal annual cash flows. These equal annual cash flows were given by S* in figure 7. The internal rate of return of any project is

$$r = \frac{S^*}{C_0} - \frac{S^*}{C_0 (1 + r)^{50}}$$

It was shown previously that for values of r in excess of 5 %, the ratio S^*/C_0 is a good estimate of the internal rate of return. The reciprocal of this ratio, that is $\frac{C_0}{S^*}$ is the project's payback period or the number of years taken to recoup the original outlay. Therefore, the reciprocal of the payback period for any fifty year project is a good estimate of that proposal's internal rate of return.[107]

The task of deriving a predetermined cut-off rate where the firm has not used discounted cash flow methods in the past can make use of this relationship between the payback reciprocal and the internal rate of return. If in a past instance of single period capital rationing the firm had available approximately the same level of funds as at present, and all projects were analysed over a fifty-year time span and were of comparable profitability to current year projects, then the payback reciprocal of the project just accepted in that prior year may provide a good estimate of the predetermined cut-off rate in the current period.[108] It is satisfactory provided this project just accepted has fairly even cash flows.

If the project just accepted during the previous instance of single-period rationing does not meet the latter requirement, then the current predetermined cut-off rate is not the payback reciprocal of this project. However, the estimate provided by this ratio should be used as a first approximation of the correct predetermined rate. Factors such as the life of the proposal and the irregularity of cash flows are considered in estimating the required cut-off rate. For instance, if the early cash flows from the project were fairly high compared with the project's expected returns in later years, then the payback period should be lengthened. The resulting estimate of the internal rate of return would be lower than if the straight payback reciprocal was used. The objective in performing these manipulations is to find the payback period that would result from the project if it had equal annual cash flows over a very long period.

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106 See above, p. 112.

107 For further information concerning the relationship between the payback reciprocal and the internal rate of return see Myron J. Gordon, "The Payoff Period and the Rate of Profit", *Journal of Business*, 28 (October 1955): 253-60.

108 The reason for taking the payback reciprocal on the project just *accepted* rather than just rejected in the past has been explained in the discussion of predetermined cut-off rates for two-period projects. See above, p. 112.

It may be argued that the levels of available funds, the relative profitability of investment projects and the time span for evaluation in prior periods of capital rationing could differ from their current year counterparts. If this occurs, the internal rate of return of the project just accepted in the past or its payback reciprocal as the case may be, would not be the correct predetermined cut-off rate. In most sophisticated businesses it is probable that long-range corporate planning extends up to a fixed period in the future, such as the fifty-year horizon used in this study. All planning in prior years would cover a time span of equal length to the current planning period. Therefore, projects in prior years may be assumed to be evaluated over the same life span as current year projects.

However, there is no easy way around the problem posed where levels of funds and project profitabilities currently applicable differ from their prior year equivalents. As pointed out earlier, the levels of investment resources and the relative profitabilities of available projects determine the shape and position of the investment opportunity curve. In using cut-off rates in prior capital rationing periods as a guide to the current predetermined rate, it is assumed that the productive opportunity curve in prior years resembles its current equivalent. This it will not do where the level of funds and profitabilities of current investment projects do not resemble their past counterparts. In these circumstances, reliance upon cut-off rates in prior years should be viewed with suspicion.

The only answer thought possible would be to allow management to weigh up the changes in profitability and volume of investment funds and the probable effect they might have on the cut-off rate. For instance, where the profitability of projects has increased and the volume of funds has not changed, the new cut-off rate should exceed the old. The cut-off rate last year should be used as a lower limit to the current year rate, despite the fact that last year the firm did not suffer from capital rationing. In many instances where all projects are divisible, project profitabilities may only differ slightly from year to year. If the cut-off rate last year was 10%, the current year cut-off rate must exceed 10% as the volume of available funds is restricted.

The cut-off rate under the maximization of ordinary shareholders' wealth

Where the goal of the firm is the maximization of the wealth of its ordinary shareholders, derivation of the correct cut-off rate proceeds in a similar fashion to the derivation of this rate for two-period projects.[109] A diagram such as figure 8 can be used for this purpose.

Ordinary shareholders' wealth is maximized by choosing that combination of current dividends and multi-period investments which places them on the highest indifference curve possible. This is the indifference curve which is just tangent to the productive opportunity curve XABC. This is shown in figure 8 as the curve u_0 which is tangent to the curve XABC at the point A. The optimum investment for the firm is to accept projects along XABC from X to A. This means that XG funds are invested in multi-period investments, and the remainder GO is distributed as dividends.

The cut-off rate is the internal rate of return of the project just rejected, and is represented by the slope of the productive opportunity curve at the point A.

Predetermined cut-off rate under the maximization of ordinary shareholders' wealth

As discussed previously,[110] there is no way that this ideal cut-off rate can be obtained in practice. The reasons for this are firstly the interdependence between the

109 See above, p. 113.

110 See above, pp. 113-14.

slope of the productive opportunity curve and the cut-off rate, and secondly the non-existence of a common schedule of ordinary shareholders' time preference rates, partly represented by the indifference curve u_0.

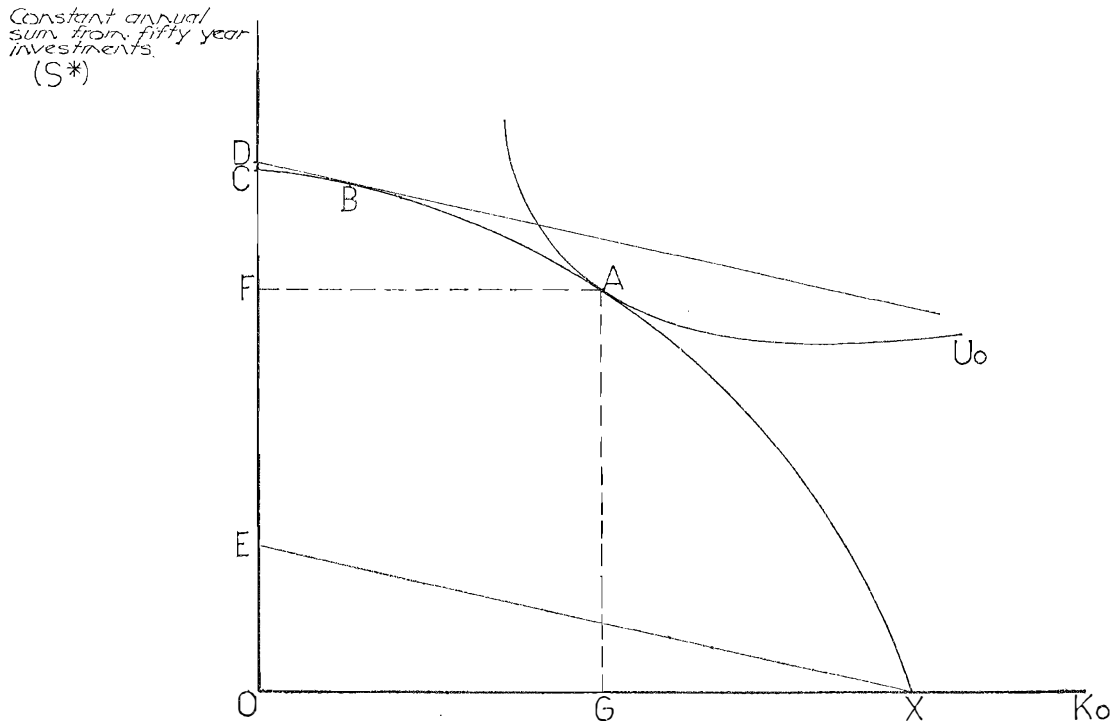

Fig. 8.—Analysis of multi-period investments under single-period rationing

Therefore, the subjective discount rate of management is suggested as an alternative. It is determined by the consumption and investment preferences of management. As in the case of two-period investments,[111] it is assumed that management has the best interests of the ordinary shareholders at heart. If it did not, then the cut-off rate determined may not maximize the wealth of these shareholders.

This subjective discount rate should be determined using as a guide the rates of return on projects just accepted in previous instances of single-period capital rationing. However, this should be subject to the following conditions. For the same reasons given in the discussion of predetermined cut-off rates for two-period projects, the volume of funds available in past instances of single-period capital rationing should approximate current funds, the projects submitted in the past should have been of comparable profitability to current multi-period investments, and the goal of management in the past should have been to maximize the wealth of the ordinary shareholders. All projects should have been evaluated over the same time period.

In the previous discussion[112] of predetermined cut-off rates under the assumption of corporate profit maximization, the situation in which some of the above conditions were not applicable was discussed. In particular, consideration was given to the problems arising when the volume of investment funds and project profitabilities in the current year differed from their prior year equivalents. These same problems are present in deriving a predetermined cut-off rate under the goal of ordinary shareholders

111 See above, p. 114.
112 See above, p. 122.

wealth maximization. The only solution that can be offered is for management to weigh up the various changes in these two variables, project profitability and level of investment funds, and to investigate the possible effects these changes could have on the cut-off rate.

Use of predetermined rates to evaluate projects

The following example will be used to illustrate how the predetermined cut-off rates derived in this section may be used to evaluate a multi-period project coming to the attention of management in a single capital rationing period. Suppose that the cut-off rate applicable in the capital rationing period has been determined as 15%. In all non-rationing periods the cut-off rate equals the cost of capital and is 10%. The goal of the firm does not matter for this illustration; it could be either corporate profit maximization or maximization of ordinary shareholders' wealth.

A project with an economic life of five years is to be evaluated. However, since the planning horizon of the firm is fifty years and all projects under consideration are being evaluated over this period, it has been estimated that the cash inflows from the project can be reinvested in the firm to return $10 every year up to the planning horizon. The resultant net cash flow pattern of the proposal takes the following form.[113]

TABLE 11

Net cash flows from five-year project

Year	0	1	2	3	4	5	6	.....	49	50
Cut-off rate	15%	10%	10%	10%	10%	10%	10%	.....	10%	10%
Net cash flow	($1081–19)	$100	$1000	$300	$100	$100	$10	.....	$10	$10

Note: All cash flows are assumed to occur at year-end.

The internal rate of return for this project is 19%, computed in table 12.

TABLE 12

Internal rate of return for five-year project

Year	Net cash flow	Discount factor at 19%	Present value
	$		$
0	(1081–19)		(1081–19)
1	100–00	.840336	84–03
2	1000–00	.706165	706–16
3	300–00	.593416	178–02
4	100–00	.498669	49–87
5	100–00	.352142	
6	10-00	.	
.	.	.	
.	.	.	63–11
.	.	.	
.	.	.	
.	.	.	
.	.	.	
50	10-00	.000167	
		Net present value	Nil

113 Discount factors are obtained from Merrett and Sykes, *The Finance and Analysis of Capital Projects*, pp. 510-29.

With a cut-off rate of 15% in the current year, a 19% internal rate of return signifies project acceptance.

The net present value of this proposal is evaluated by discounting the net cash flows in each year to their present values in year zero, using as discount rates the cut-off rate applicable in each year. This gives a net present value of $195-22 as shown in table 13. This positive net present value again signifies project acceptance. In addition, the present value index is $\frac{1276\text{-}41}{1081\text{-}19}$ or 1.18, also an indication that the project should be accepted.

TABLE 13

Net present value of five-year project

Year	Net cash flow	Discount factors at 10%	Present value of cash inflows at year 1	Present value of all cash flows at year zero
	$			$
0	(1081–19)			(1081–19)
1	100–00	1.000000	100–00	
2	1000–00	.909091	909–09	
3	300–00	.826446	247–93	
4	100–00	.751315	75–13	
5	100–00	.683013		
6	10–00	.620921		
.	.	.		
.	.	.	135–66	
.	.	.		
.	.	.		
.	.	.		
.	.	.		
50	10–00	.009370		
Discount factor at 15%		.869565	1467–81	1276–41
			Net present value	$195–22

It is of interest to note the following difficulty experienced by the internal rate of return criterion. Suppose that capital rationing is not experienced in period zero as before, but is now imposed on the firm in period one. The cut-off rate in period zero returns to 10%, and because very stringent capital rationing is imposed in period one, the cut-off rate in the latter year soars to 50%. All other cut-off rates are unchanged. In these circumstances, the internal rate of return for the project does not change, being 19% as computed in table 12. With a period zero cut-off rate of 10% the project is still acceptable using this criterion.

However, the net present value and present value index criteria tell a different story. From table 14, the net present value of the project is now—$55.46. The present value index is $\frac{1025\text{–}72}{1081\text{–}19}$ or .95. The figures for both these criteria signify that the project should be rejected.

The conflict in accept or reject decisions arises from the different assumptions underlying each criterion. With the internal rate of return it is assumed either that capital outstanding in the project is recovered at a constant rate per annum equal to the internal rate of return, or that cash benefits from the project can be reinvested in the firm at the internal rate of return.[114] Both these assumptions are negated when reinvestments in period one must return at least 50% per year to be accepted. On

114 See above, p. 101.

the other hand, both the net present value and present value index criteria assume that cash benefits are reinvested at the cut-off rate in each year, so they both take the high period one cut-off rate into account. The decision to reject the project, given by both these criteria, is the correct one.

TABLE 14

Net present value of five-year project when capital rationing occurs in year one

Year	Net cash flow	Discount factor at 10%	Present value of cash flows in year 2	Discount factor at 50%	Present value of cash flows in year 1	Discount factor at 10%	Present value of cash flows in year zero
	$		$				$
0	(1081–19)					1.000000	(1081–19)
1	100–00			1.000000	100–00		
1	1000–00	1.000000	1000–00				
3	300–00	.909091	272–73				
4	100–00	.826446	82–64				
5	100–00	.751315					
6	10–00	.683013					
.	.	.					
.	.	.					
.	.	.	187–25				
.	.	.					
.	.	.					
.	.	.					
50	10–00	.010307					
			1542–62	.666667	1028–41		
					1128–41	.909091	1025–72
						Net present value	($55–46)

The conclusion that can be drawn from this example is that the internal rate of return should not be trusted to give the correct accept/reject decision where the firm is likely to experience capital rationing in single future periods.

Further complications arising from single-period rationing

Even though the cut-off rates for single-period rationing have been found, the difficulties caused by this restriction are not completely solved. Because capital funds are limited in the current period only, management should consider the possibilty of postponing some projects until later years when they may be undertaken quite freely. Essential projects which meet the required standards of acceptability could be undertaken during the single rationing period, and other projects whose profitability is not likely to suffer because of postponement could be deferred. In this way, the long-term benefits to the firm will be maximized.[115]

Summary of analysis for single-period rationing

In future periods when all restrictions on available funds are lifted, there are identical cut-off rates applicable under either the goal of corporate profit maximization or the goal of maximizing ordinary shareholders' wealth. These rates are the

115 For a discussion of project postponement under these conditions see Stephen A. Marglin, *Approaches to Dynamic Investment Planning* (Amsterdam: North-Holland Publishing Co., 1963), pp. 37-62.

firm's cost of capital in these future years. The cut-off rates derived for the current period when single-period rationing applies, are summarized in table 15.

TABLE 15
Summary of first year cut-off rates for multi-period investments under single-period rationing

Goal of firm	Theoretical cut-off rate	Predetermined cut-off rate
Corporate profit maximization	Internal rate of return of the project just rejected	Payback reciprocal on project just accepted in previous single rationing period
Maximization of ordinary shareholders' wealth	Internal rate of return of project lying at the point of tangency of productive opportunity curve and shareholders' indifference curves	Subjective discount rate of management

Multi-period rationing

If restrictions on investment funds are imposed over several periods, the timing and magnitude of future cash flows from current investments becomes very important. Not only do these future cash flows play a role in determining projects' acceptability now, but also their timing and magnitude can help to determine the volume of funds available in future periods for investments and dividend payments.

The cut-off rates in future years can be used to determine which projects are currently accepted. But these cut-off rates depend on the volume of funds available in the future and this depends on the proposals which are accepted now. This circularity is the essence of the problem involved in computing cut-off rates under multi-period rationing.

> In planning a multiperiod investment programme the fact that the time-series of discount rates for proposals adopted in period one depends upon the programmes undertaken in every subsequent year might suggest that the programme should be planned backwards from the terminal period to the initial period. Unfortunately this approach encounters the further difficulty that the financial constraints ruling in the later periods of a multiperiod problem may be a function of the cash-flows of the projects undertaken in the earlier periods.[116]

In addition to this difficulty, the cut-off rates for future periods are likely to fluctuate quite significantly because of the multi-period funds constraint. The firm has no access to the external capital market during these periods, and sole reliance on internal generation may cause wide fluctuations in the volume of funds available each year. This causes variations in future cut-off rates.

> To assume a constant cost of capital over future years is a less tenable assumption under capital rationing conditions than under normal conditions. This is especially so when capital rationing takes the form of raising no further capital at all, relying entirely on self generated funds, i.e. depreciation provisions and retained profits. These latter can fluctuate significantly from year to year. To calculate the marginal cost of capital in such circumstances, however, involves forecasting all likely investment projects for many years ahead, together with all the funds which they in turn would generate.[117]

116 Lawson and Windle, *Capital Budgeting*, p. 60.

117 Merrett and Sykes, *The Finance and Analysis of Capital Projects*, p. 141. They use the term "cost of capital" in the same sense as the term "cut-off rate" has been used in this study.

The discounting methods discussed in this study are useless in the face of this multi-period funds restriction, "a problem which is pervaded by some extremely complicated feed-backs and interrelationships".[118] Mathematical programming techniques represent the only means by which project selection under these conditions can be achieved. However, these techniques are beyond the scope of this study and will not be pursued further.[119]

The rationale behind multi-period rationing and the methods of project selection used to cope with it must be questioned when this funds restriction is internally imposed. It was shown earlier[120] that the continual imposition of funds restrictions on the firm is contrary to the objective of profit-making. Both the firm and its ordinary shareholders suffer because this imposition starves the firm of otherwise profitable projects over a long period, and its prosperity and future growth are effectively cramped. The use of mathematical programming techniques to ration funds in these circumstances does not really overcome the basic problem facing the firm. Management should realise that their actions in restricting funds are affecting both the firm and its ordinary shareholders, and the funds limit which is retarding the prosperity of the firm should be lifted.

> The imposition of a succession of financial constraints upon successive budgeting periods is to compromise the *raison d'être* of business enterprise—the profit-making motive. Multiperiod self-imposed capital rationing is thus a policy of second best. The very notion of using sophisticated programming techniques to achieve the objectives of a policy of second best is tantamount to setting the objective of doing the wrongs things very well.[121]

Empirical results

Having discussed at some length the cut-off rates for multi-period investments under capital rationing, an examination will now be made of current practice in Australia to observe how businessmen actually go about adjusting cut-off rates under capital rationing. Evidence is obtained from the survey of listed Australian public companies carried out by the writer.

However, before a detailed analysis of the findings of the survey are given, it is beneficial to note the number of companies using discounted cash flow methods, because many respondents not using these techniques indicated that they too set minimum cut-off rates for use with their arbitrary methods of capital expenditure analysis. Most information can be gained from the empirical data if it is divided into two sections: those firms that do use discounting methods and those that do not.

Companies in the sample were asked:

1. Does your firm use discounted cash flow techniques in capital expenditure analysis?

   1. Yes
   2. No

Forty-two companies answered yes to this question, but from the qualifications attached to many of these responses, discounted cash flow is not used all the time by all these firms. This response represented 24.4% of the sample.[122]

A prerequisite to ascertaining whether companies modify their cut-off rates when funds are rationed is to discover whether they set a cut-off rate when funds are unlimited. The following question was asked:

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118 Lawson and Windle, *Capital Budgeting*, p. 60.

119 Readers interested in the mathematical programming approach to capital rationing are referred to Baumol and Quandt, "Investment and Discount Rates".

120 See above, p. 98.

121 Lawson and Windle, *Capital Budgeting*, pp. 60-61.

122 The 95% confidence limits for the proportion of the population using discounted cash flow techniques were computed to be 20% and 31%.

4. Does your Company *ever* set a minimum rate of return % standard for proposal acceptance?

1. Yes
2. No

The results of this question are summarized in table 16.[123]

TABLE 16

Analysis of cut-off rates when funds are unlimited

	Use discounted cash flow		Do not use discounted cash flow	
	Number	Percentage	Number	Percentage
Set a minimum rate	36	85.7	45	36.6
Do not set a minimum rate	6	14.3	78	63.4
Total	42	100.0	123	100.0

It is interesting to observe that 85.7% of those companies using discounted cash flow methods set a minimum cut-off rate. This is far higher than the corresponding proportion for companies not using discounted methods.

Those companies setting a minimum cut-off rate were then asked whether this rate *increased* when capital rationing was imposed. The specific question was:

5. If the answer to question 4 is "yes," and your firm is always or occasionally short of funds for capital expenditure, does this minimum rate of return ever increase when funds are short?

1. Yes
2. No

The results from this question[124] are summarized in table 17.

TABLE 17

Variations in the minimum cut-off rate when funds are rationed

	Use discounted cash flow		Do not use discounted flow	
	Number	Percentage	Number	Percentage
Increase minimum cut-off rate	4	14.3	7	17.1
Do not increase minimum cut-off rate	24	85.7	34	82.9
Total	28	100.0	41	100.0

123 Note that all companies in the sample did not answer this question.

124 Only 69 of the 81 companies in the sample that do set a minimum cut-off rate answered this question.

Regardless of whether discounted cash flow methods are used or not, it would appear that very few firms increase their minimum cut-off rates when funds are short. Those companies that do were asked to outline how they determined the new rate. Replies were received from three companies which use discounting methods. One company stated that the new rate was determined "very arbitrarily"; another said that "only the most profitable projects are considered"; while the third stated "preference is shown to projects showing better than standard % return, all other aspects being equal".

These replies are not very heartening and would suggest that firms lack knowledge of theoretical methods for dealing with rationed funds.

Some companies using discounting indicated that they did not increase their minimum rate of return but tended to concentrate on projects which had reduced elements of risk. This was evidenced in two replies received. One stated that the company tried to reduce risk factors in the proposals being considered; the other indicated that it switched from discounting to payback as a capital expenditure evaluation method when funds were short. Risk factors are crucial in any capital project and their elimination is commendable especially when funds are tight, but since this study only considered projects with equal or negligible degrees of risk, this aspect was not given any consideration in the previous theoretical discussion.

Conclusions

In this section, it has been demonstrated that the analysis developed for two-period investments in section III may be extended to encompass multi-period investments under single-period rationing. The cash flows for any project should be discounted to present values using a series of cut-off rates: those rates applying in future years, and the rate applying in the current period when single-period rationing is imposed. Future cut-off rates are equal to the firm's cost of capital although its measure will depend on whether the goal of the firm is corporate profit maximization or maximization of ordinary shareholders' wealth. The cut-off rate in the current period also depends on which of these goals is applicable. Care should be taken when single period rationing occurs in a future year. The internal rate of return criterion may not give the correct accept/reject decision for a project in this case, and the net present value of present value index criteria are preferred.

Multi-period rationing is likely to apply in some governmental organizations where the availability of funds over many years depends on appropriations from a central treasury. In these cases, use of sophisticated mathematical techniques for allocating resources among projects is justified. However, such methods should not be used where the funds restriction is internally imposed.

It was disappointing to discover that public companies in Australia seem to lack knowledge of correct techniques to cope with capital rationing. Since capital rationing is so prevalent in the business environment, one would hope that progress in this area is made rapidly. From results of the survey performed by this writer, it can only be concluded that, in all probability, incorrect investment decisions are currently being made.

V. CONCLUSIONS

A capital rationing situation has been defined as one in which the firm either cannot or does not wish to raise sufficient funds for all capital expenditure proposals deemed profitable. It was the aim of this study to examine the behaviour of cut-off rates in capital expenditure analyses when a firm is subjected to capital rationing. Usefulness in ascertaining project acceptability constitutes the main function of cut-off rates. They are used as the minimum acceptable rates of return for comparisons with the internal rate of return criterion, and as the chosen rates of discount in com-

puting net present value and the present value index. However, before this examination of cut-off rates could be carried out, it was necessary to discover whether firms are currently suffering capital rationing, since there is no point in describing detailed theoretical techniques to deal with something that does not exist.

From a survey carried out by the writer, it was found that 87.2% of the listed Australian public companies in the sample experienced capital rationing, the 95% confidence interval for the percentage of all similarly affected listed public companies being 81.8% to 91.2%. To a great extent, this restriction on funds was imposed by management. Of the companies sampled 83.7% experienced this internal capital rationing. On the other hand, external capital rationing, or investment restriction from outside the firm, was experienced by only 39.5% of the sample.

Because of these findings, an analysis of cut-off rates under capital rationing was thought to be warranted. This analysis was commenced in section III and was developed in section IV. After presentation and criticism of the "conventional analysis" of cut-off rates under capital rationing, two-period investments under either external or internal rationing were introduced. By focussing on investments which terminated after two years, the aim was to isolate one issue overlooked by the conventional analysis. This was the fact that differing goals of the firm could alter the cut-off rate for capital projects and the volume of funds available for dividend payments. A firm under either external or internal capital rationing has to decide how much of its limited funds to make available for capital expenditures and how much to pay out as dividends. Dividend payments and levels of capital expenditure are closely linked where funds are rationed since an increase in one automatically leads to a reduction in the other. Volumes of funds made available for each of these tasks differ depending on whether the goal of the firm is maximization of the wealth of its ordinary shareholders or corporate profit maximization. Once the volume of investment funds has been determined under these goals, the analysis in section III demonstrated how completely separate cut-off rates applied for each goal. This resulted in different projects being accepted in each case. Where all two-period investments are not known in advance, predetermined cut-off rates were introduced. Any two-period project coming to the attention of management throughout the year may be evaluated using the predetermined rate. Either the internal rate of return, net present value or present value index criteria are available for this evaluation.

It was thought that an organization having only two-period investments to consider would be an exception to the norm, and that most firms would be considering only multi-period projects. Therefore, the argument provided in section III was expanded to cater for this situation. In so doing, several issues overlooked in the conventional analysis were discussed. These were the implications of single-period and multi-period rationing, and equating the useful lives of projects under consideration. For a situation of single-period rationing, theoretically correct cut-off rates and their predetermined equivalents were derived under either the goal of corporate profit maximization or the goal of ordinary shareholders' wealth maximization. However, these cut-off rates should be used only with the net present value or the present value index criteria. The optimum investment decision can be made by reducing cash flows to their present values using the cost of capital as discount factor in all non-rationing periods and the cut-off rates determined in section IV for the period in which capital rationing applies.

In deriving these cut-off rates, all projects were compared over the life of the longest investment considered, in this case fifty years. This necessitated projections of reinvested cash flows from shorter lived projects over the interim period. These projections would need to be derived from long-range planning activities within the organization. The advantage of this comparison over fifty years is that all cash flows attributable to current projects and their reinvested proceeds can be compared on equal terms and no unfair advantage is given to long-lived projects with resultant bias to the cut-off rate.

Where capital rationing is imposed over several periods, this study cannot provide any cut-off rates for capital expenditure evaluation. So many complicated feed-backs and inter-relationships exist between the future cut-off rates and the batch of projects presently under consideration that the only correct means of selection is to use mathematical programming techniques. This is only recommended in cases where rationing is externally imposed. Relief from this fund limitation is largely beyond the control of firms since the underlying cause is market imperfections. Management is justified in attempting to achieve the best allocation of limited funds in this case. Where multi-period rationing is internally imposed, use of these sophisticated techniques is not justified. If the goal of the firm is to maximize corporate profits or to maximize the wealth of its ordinary shareholders, multi-period internal capital rationing is of doubtful logic. Such a policy will never lead to the attainment of these profitability goals and will only stifle the potential growth and prosperity of the firm. Of course, if internal capital rationing is imposed in one period only, and there could be justifiable reasons for this, the cut-off rates developed in section IV may be used to arrive at the optimum combination of projects.

The analysis presented in this study should be subject to two qualifications. Firstly, the assumption was made that all projects are divisible and independent. This resulted in the smooth demand curves for capital in the conventional analysis, and the smooth productive opportunity curves drawn in the analysis of cut-off rates with two-period and multi-period projects. Where this assumption does not hold, the necessity might arise for a ranking of projects to obtain the set of investments that best uses the funds available. This aspect of project selection has not been considered. Secondly, a significant aspect of project evaluation is the degree of risk attached to expected future cash flows. Further research is needed to determine the effect on cut-off rates under capital rationing in these circumstances.

Capital rationing is a phenomenon which affects many businesses. Although some cannot escape from it and are continually forced to make investment decisions in its shadow, there are other companies where internally imposed restrictions could be lifted so that every profitable investment opportunity is utilized. Firms in the latter circumstances are urged to relax their fund restrictions. For organizations where capital rationing is imposed either externally or internally in isolated single periods, it is hoped that the cut-off rates derived in this study are used to achieve the best allocation of limited resources.

APPENDIX I

Source data

The population data for the survey of listed Australian public companies come from stock exchange listings of companies in Brisbane, Sydney, Melbourne, Adelaide, Perth and Hobart, at 30 June 1969. Before these data can be used, however, it is necessary to eliminate all multiple listings of companies, so that each appears once. The reason for this is that each company in the population should have an equal chance of selection. A firm listed in both Sydney and Melbourne, for instance, would have twice the normal chance of selection and so this double listing must be eliminated.

Sample size

The main requirement of the sample is to discover the percentage of listed public companies experiencing capital rationing. If it could be assumed that n samples are taken from the population, and a note made of the percentage in each experiencing capital rationing, the statistical distribution of these percentages would approximate a

normal distribution.[125] The mean of the distribution of sample percentages is π (the true population percentage), and the standard derivation is σ_p (the standard error).

The relationship between the standard error and the true population percentage is given by the following equation:

$$\sigma_p = \sqrt{\frac{\pi(1 - \pi)}{n}}$$

where n is the required sample size.

The 95% confidence interval for π is $\pi \pm 1.96\,\sigma p$. If a sample proportion p, is obtained from this distribution, it can be stated that the true population lies inside the limits $p \pm 1.96\,p$ with 95% confidence.[126]

To obtain a sample size, the judgment of the investigator must be exercised. A maximum tolerable derivation from the true population percentage must be stipulated That is, $1.96\sigma p$ must be given a specific value. The writer chose .075.

$$\therefore\ 1.96\sigma_p = .075$$

$$\sigma_p = \frac{.075}{1.96}$$

$$\text{Now, } \sigma_p = \sqrt{\frac{\pi(1 - \pi)}{n}}$$

$$\frac{(.075)^2}{(1.96)^2} = \frac{\pi(1 - \pi)}{n}$$

$$n = \frac{\pi(1 - \pi)}{\frac{(.075)^2}{(1.96)^2}}$$

To obtain the largest sample size necessary in these circumstances, it is necessary to impute values for π and $1 - \pi$ that maximize the numerator of this equation. The expression $\pi(1 - \pi)$ is maximized only when $\pi = .5$

$$n_{max} = \frac{.5 \cdot .5}{\frac{(.075)^2}{(1.96)^2}}$$

$$= 172.$$

Therefore, the sample size taken for this survey is 172.

The sample is stratified into 7 industry groups using the following equation:[127]

$$\frac{\text{Number of Companies in Stratum A}}{\text{Total Companies in Population}} \cdot \text{sample size} = \text{Number of Companies in the Sample from Stratum A.}$$

The numbers of companies in each stratum are shown in table 18.

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125 This occurs by virtue of the Central Limit Theorem. See Brunk, *An Introduction to Mathematical Statistics*, pp. 164-69.

126 See Croxton and Cowden, *Applied General Statistics*, pp. 671-75.

127 It should be appreciated that this method of stratification is not the most accurate available. Better methods use a weighted average ratio of companies in one stratum to the total population. The weighting factor is the standard deviation in each population stratum. However, since these standard deviations in the strata are not known, the method actually used is satisfactory.

TABLE 18
Number of companies sampled from each stratum

Stratum	Number in population	Number in sample
Finance	150	19
Heavy Industry	203	25
Light Manufacturing	390	48
Primary Industry	58	7
Trade and Service	414	51
Mining	132	16
Oil	48	6
Total	1,395	172

Using a set of random number tables,[128] the sample of 172 companies is selected from these strata.

Confidence limits for π

It was stated in section II[129] that the percentage of companies in the sample suffering from capital rationing was 87.2. To obtain the 95% confidence limits for π, the percentage of *all* listed Australian public companies experiencing capital rationing, the following formula may be used:

$$Z_{.975} = \frac{p - \pi}{\sqrt{\frac{\pi - \pi^2}{n} \cdot \frac{N - n}{N - 1}}}$$

Where: $Z_{.975}$ = the X co-ordinate of a standard normal distribution, cutting-off an area of .025 in one "tail" of this distribution.
p = sample percentage
π = population percentage
N = total number of companies in population
n = number in sample.
$\frac{N - n}{N - 1}$ = finite population correction

$$1.96 = \frac{.872 - \pi}{\sqrt{\frac{\pi - \pi^2}{172} \cdot \frac{1223}{1394}}}$$

Simplifying this equation gives a quadratic function of π. When solved, this gives the limits for π as 81.8% and 91.2%.

128 See Brunk, *An Introduction to Mathematical Statistics*, pp. 402-406.
129 See above, p. 85.

APPENDIX II

University of Queensland
Department of Accountancy
SURVEY OF CAPITAL RATIONING IN AUSTRALIAN PUBLIC COMPANIES

Company No. *Classification*

Please (a) Use a tick mark ($\surd$) to indicate the appropriate alternative; and
(b) Supply details where requested.

ALL INFORMATION OBTAINED WILL BE TREATED IN THE STRICTEST CONFIDENCE. ONLY *AGGREGATED* RESULTS FROM ALL COMPANIES USED.

---

1. Does your firm use discounted cash flow techniques in capital expenditure analysis?
   1. Yes
   2. No
2. In any one capital budget period has the supply of funds been sufficient to finance all the acceptable (profitable) proposals submitted for consideration?
   (1) *Always* sufficient funds available for proposals.
   (2) *Occasionally* capital funds *NOT* sufficient for proposals.
   (3) Never sufficient funds for all proposals submitted.
3. Has management ever ruled that total funds allocated to capital projects be limited to a *predetermined amount*?
   1. Regularly
   2. Occasionally
   3. Never
4. Does your Company *ever* set a minimum rate of return % standard for proposal acceptance?
   1. Yes
   2. No
5. If the answer to question 4 is "yes," and your firm is always or occasionally short of funds for capital expenditure, does this minimum rate of return ever increase when funds are short?
   1. Yes
   2. No
6. If you answered "yes" to question 5, outline how you determine the new rate

   .....

   .....

   .....

7. Is capital rationing in your Company:
   (1) Caused by a desire of management to limit funds for capital expenditure proposals?
   (2) Caused by restrictions being placed on the Company by external bodies?
   (3) Caused by both these factors?

8. If capital rationing is caused by external factors, are these
   (1) Difficulties in obtaining funds from
      (a) banks and short-term creditors
      (b) the issue of debentures
      (c) the issue of new shares?
   (2) Restrictions imposed on the firm by Govt. authority?
   (3) Restrictions imposed by existing debenture trust deeds?
   (4) Other reasons (specify).....
   .....
9. Does your company limit funds available for capital expenditure proposals in specific areas and not others?
   1. Yes
   2. No
   3. Sometimes
10. If the answer to question 9 is "Yes" or "Sometimes," are funds limited in the case of:
   (1) Expansionary proposals Company-wide (e.g. buying a new factory)?
   (2) Replacement proposals Company-wide?
   (3) Expansionary proposals in a segment of the Company?
   (4) Replacement proposals in a segment of the Company?

   Thank you for co-operating in this survey.

   Any comments (if you so desire).

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